High order mimetic discretization

A. Palha[1]

[1] *Delft University of Technology, The Netherlands* a.palha@tudelft.nl

In this work the High Order Mimetic Discretization Framework will be presented together with a discussion of two fundamental aspects for the construction of structure-preserving discretizations: (i) the definition of the discrete degrees of freedom of physical field quantities, and (ii) the formulation of the physical field laws.

For the first, the geometric degrees of freedom will be introduced. These degrees of freedom are associated to geometric objects (points, lines, surfaces and volumes), and a relation to differential forms will be remarked. It will be shown that it is possible to construct discrete polynomial function spaces of arbitrary degree associated to these geometric degrees of freedom. Moreover, these function spaces constitute a discrete de Rham complex:

$$\mathbb{R} \longrightarrow V_h^0 \subseteq H(\nabla, \Omega) \xrightarrow{\nabla} V_h^1 \subseteq H(\nabla\times, \Omega) \xrightarrow{\nabla\times} V_h^2 \subseteq H(\nabla\cdot, \Omega) \xrightarrow{\nabla\cdot} V_h^3 \subseteq L^2(\Omega) \longrightarrow 0\,.$$

In this way it is possible to exactly discretize topological equations even on highly deformed meshes. All approximation errors are included in the constitutive equations. This leads to discretizations that exactly preserve the divergence free constraint of velocity fields in incompressible flow problems and of magnetic fields in electromagnetic problems, for example.

For the second, the Navier-Stokes equations will be used as an example and we will show that although at the continuous level all equivalent formulations are equally good, at the discrete level, the choice of a particular formulation has a fundamental impact on the conservation properties of the discretization.

These ideas will be illustrated with the application to the solution of: e.g. Poisson equation [1], Darcy flow, fusion plasma equilibrium [2], and Navier-Stokes equations [3].

For the efficient construction and solution of mimetic discretizations a hybrid algebraic dual formulation will be presented. This formulation is based on broken Sobolev spaces together with specially constructed test functions. The use of broken Sobolev spaces separates the degrees of freedom into interior degrees of freedom and interface (boundary) degrees of freedom. This in turn enables the use of efficient static condensation for the solution of the system of equations. The use of algebraic dual test functions gives rise to a much sparser system of equations and eliminates further the presence of metric dependent quantities.

References

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*University of Franche-Comté Seminar: High order mimetic discretization