OPTIMAL AND DUAL STABILITY RESULTS FOR L^1 VISCOSITY AND L^∞ ENTROPY SOLUTIONS

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ABSTRACT. We revisit stability results for two model examples of nonlinear degenerate parabolic PDEs, one being fully nonlinear and the other in divergence form. We consider more precisely the HJB equation

$$\partial_t \varphi = \sup_\xi \{b(\xi) \cdot D\varphi + \operatorname{tr}(a(\xi) D^2 \varphi)\}$$

and the anisotropic degenerate parabolic equation

$$\partial_t u + \operatorname{div} F(u) = \operatorname{div}(A(u)Du).$$

Their solutions are interpreted in the viscosity and entropy senses, and they satisfy contraction principles in L^∞ and L^1 respectively. Our aim is to get similar properties for L^1 viscosity and L^∞ entropy solutions. For the first equation, L^1 is in fact too weak and we identify the Banach topology which is just stronger enough to have stability. This gives us an optimal L^1 type Banach framework in which we obtain a general quasicontraction principle. For the second equation, we propose a new weighted L^1 contraction principle allowing for pure L^∞ solutions. Our main contribution is to show that the solutions of the HJB equation can be used as weights and that this choice is optimal. Interestingly, this reveals a new type of duality between entropy and viscosity solutions.