

**MODELLING PHASE SEPARATION WITH COUPLED ELLIPTIC
EQUATIONS: RECENT RESULTS ON THE ASYMPTOTIC
ANALYSIS**

ALESSANDRO ZILIO

We consider a family of positive solutions to the system of k components

$$-\Delta u_{i,\beta} = f(x, u_{i,\beta}) - \beta u_{i,\beta} \sum_{j \neq i} a_{ij} u_{j,\beta}^2 \quad \text{in } \Omega,$$

where $\Omega \subset \mathbb{R}^N$ with $N \geq 2$. It is known that uniform bounds in L^∞ of $\{\mathbf{u}_\beta\}$ imply convergence of the densities to a segregated configuration, as the competition parameter β diverges to $+\infty$. In this talk I will discuss how to obtain sharp quantitative point-wise estimates for the densities around the interface between different components, and, more specifically, how to characterize the asymptotic profile of \mathbf{u}_β in terms of entire solutions to the limit system

$$\Delta U_i = U_i \sum_{j \neq i} a_{ij} U_j^2.$$

These results are part of an ongoing project with Nicola Soave.