Notions of solution for conservation laws with fractional diffusion

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The talk is devoted to different, complementary solution theories for equations under the form

$$u_t + \mathrm{div} f(u) + \mathcal{L}[u] = 0$$

where $\mathcal{L}$ is a fractional diffusion operator of order $0 < \alpha < 2$.

We are particularly interested in the range $0 < \alpha < 1$ where the problem inherits the main features of hyperbolic first-order conservation laws. After recalling the idea and the technical issues related to Alibaud's definition of entropy solution in L^∞ framework, three different results will be presented.

First, we will explain the construction that led to non-uniqueness result for merely weak solutions. This further established the importance of the entropy formulation. Second, we propose a notion of kinetic solution and discuss the associated techniques, that turn out to be somewhat more direct than the entropy techniques (in particular, the cutting of singularity of the non-local diffusion operator can be avoided); as a result, we get a pure L^1 well-posedness theory. Third, we exhibit the generalization of renormalization techniques to non-local diffusion operators; this leads to $L^1 + L^\infty$ well-posedness theory.

The talk is based upon joint works with Nathaël Alibaud (Besançon), Mostafa Bendahmane (Bordeaux), and Adama Ouedraogo (Bobo-Dioulasso, Burkina-Faso).