Erratum to : Revising Uniqueness for a Nonlinear Diffusion-Convection Equation

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Abstract

The proof of [1, Lemma 3] was incomplete. Here we give the missing arguments, under very weak regularity assumptions on the domain Ω coming from the paper [2] of the authors.

First, note that the factor $\psi = \psi(t)$ was forgotten in most of the terms that figure in the proof of [1, Lemma 3]; all the integrals that do not contain ψ_t or $\psi(0,\cdot)$ should contain the factor ψ.

Next, in the proof of [1, Lemma 3, p.75], we studied the term

$$I^2_{n,\varepsilon} = \iint_Q \mathcal{F}_\varepsilon \cdot \nabla(\xi(1-\xi_n))\psi,$$

where, with the notation of [1],

$$\begin{aligned}\mathcal{F}_\varepsilon &= \int_0^w (F(j(\varphi_0^{-1}(r)), r) - F(j(k), \varphi(k))\, H'_\varepsilon(r - \varphi(k))\, dr \\ &= \frac{1}{\varepsilon}\int_{\min(w,\varphi(k))}^{\max(w,\varphi(k)+\varepsilon)} (F(j(\varphi_0^{-1}(r)), r) - F(j(k), \varphi(k))\, dr.\end{aligned}$$

The next point of our proof in [1] was that

$$\lim_{\varepsilon\to 0} I^2_{n,\varepsilon} = 0; \tag{1}$$

yet the statement (1) is not exact. We point out that the lemma was stated under the assumption

$$F(u,w) = F_1(w) + u\, F_2(w) \text{ with } F_i \in \mathcal{C}(\mathbb{R};\mathbb{R}^N) \text{ and } F_2(0) = 0.$$

The above requirement $F_2(0) = 0$ and the homogeneous boundary condition for $w = \varphi(v)$ on $(0,T)\times\partial\Omega$ are the crucial properties needed to complete the proof of [1, Lemma 3].

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We now rectify the bound on $\mathcal{F}_\varepsilon$. In general, in the place of (1) we have

$$\lim_{\varepsilon\to 0} |\mathcal{F}_\varepsilon| \le \Big(j(\varphi_0^{-1}(\varphi(k){+}0)) - j(k) \Big) \max\{|F_2(w)|, |F_2(\varphi(k))|\}; \tag{2}$$

here $\Big(j(\varphi_0^{-1}(\varphi(k){+}0)) - j(k) \Big)$ denotes the jump at the point k of the graph $j \circ \varphi^{-1}$, φ_0^{-1} being the left-continuous inverse of φ. Notice that the jump $(j(\varphi_0^{-1}(\varphi(k)+0)) - j(k))$ is finite[1], for all $k \ge 0$.

The exact statement that replaces (1) is:

$$\lim_{n\to\infty} \lim_{\varepsilon\to 0} I^2_{n,\varepsilon} = 0. \tag{3}$$

To get (3), we exploit the techniques of [2]. Let us recall that $(\xi_n)_n$ is a sequence such that $\xi_n \in H^1_0(\Omega)$, $0 \le \xi_n \le 1$ and $\xi_n \to 1$ in $L^1(\Omega)$. In particular, the distance-to-the-boundary functions $\xi_n : x \mapsto \min\{1, n\,\mathrm{dist}(x, \partial\Omega)\}$ can be chosen. We have $\mathrm{supp}\,\nabla\xi_h \subset \Omega_{\frac{1}{n}} := \Big\{ x \in \Omega \,|\, \mathrm{dist}(x, \partial\Omega) < \frac{1}{n} \Big\}$ and

$$\frac{1}{\mathcal{M}} \le \int_\Omega |\nabla\xi_n|, \quad \int_\Omega |\nabla\xi_n|^q \le n^{q-1}\mathcal{M} \ \text{ uniformly in } n, \text{ for } 1 \le q < +\infty \tag{4}$$

(here we assume that $|\Omega_{\frac{1}{n}}| \le \mathcal{M}/n$, see hypothesis $(H1)$ and Remarks 5.1, 5.2 in [2]). Then arguing in the same way as in [2, Lemma 5.8] (the assumptions $w \in L^2(0,T; H^1_0(\Omega))$ and $F_2(0) = 0$ are exploited), we show that

$$\lim_{n\to\infty} \iint_Q |F_2(w)|\, |\nabla\xi_n|\, \xi\psi = 0. \tag{5}$$

Hence the conclusion (3) follows, and the proof of [1, Lemma 3] is complete under the additional assumptions $(H1)$, $(H2)$ from [2] on the regularity of Ω; this includes weakly Lipschitz domains and many others.

Further, in [1, Section 3], we stated that the uniqueness results of the paper remain true for nonlinear Leray-Lions kind diffusions; this is true, under the version of assumption $(H2)$ in [2] adapted to the case $p \ne 2$. The proof of (5) is adapted in a straightforward way, using (4) with $q = p'$.

References

[1] B. Andreianov and N. Igbida. Revising uniqueness for a nonlinear diffusionconvection equation. *J. Diff. Eq.* 227, pp.69–79 (2006).

[2] B. Andreianov and N. Igbida. Uniqueness for inhomogeneous Dirichlet problem for elliptic-parabolic equations. *Proc. Royal Soc. Edinburgh*, 137A, pp.1119–1133 (2007).

[1] The case where φ is constant on $[k, +\infty)$ should be excluded; in this case, $w \le \varphi(k)$ on Q, and we have $I^2_{n,\varepsilon} = 0$ for all $\varepsilon > 0$.