

# Invasion patterns in competition-ecodiffusion systems

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## Abstract

The possible control of competitive invasion by infection of the invader and multiplicative noise is studied. The basic model is the Lotka-Volterra competition system with emergent carrying capacities. Several stationary solutions of the non-infected and infected system are identified as well as parameter ranges of bistability. The latter are used for the numerical study of diffusive invasion phenomena. The Fickian diffusivities, the infection but in particular the white and colored multiplicative noise are the control parameters. It is shown that not only competition, possible infection and mobilities are important drivers of the invasive dynamics but also the noise and especially its color and the functional response of populations to the emergence of noise.

The variability of the environment can additionally be modelled by applying Fokker-Planck instead of Fickian diffusion. An interesting feature of Fokker-Planck diffusion is that for spatially varying diffusion coefficients the stationary solution is not a homogeneous distribution. Instead, the densities accumulate in regions of low diffusivity and tend to lower levels for areas of high diffusivity. Thus, the stationary distribution of the Fokker-Planck diffusion can be interpreted as a reflection of different levels of habitat quality [1–5]. The latter recalls the seminal papers on environmental density, cf. [6–7]. Appropriate examples will be presented.

## References

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