

Autumn School<sup>1</sup>  
“Multipliers in Noncommutative Analysis and their Applications”

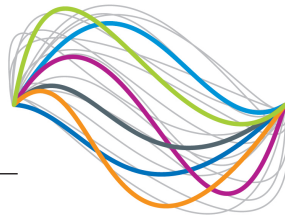
Venue: Salle 316B, LMB, Campus Bouloie, Besançon

Dates: 18 - 22 novembre 2019

Organizers: Uwe Franz, Yulia Kuznetsova, Alexandre Nou, Antonín Procházka

UBFC

UNIVERSITÉ  
BOURGOGNE FRANCHE-COMTÉ



| Time        | Monday      | Tuesday      | Wednesday   | Thursday    | Friday      |
|-------------|-------------|--------------|-------------|-------------|-------------|
| 9:00-10:00  | Parcet      | Ricard       | de la Salle | Parcet      | Ricard      |
| 10:00-10:30 | Coffee      | Coffee       | Coffee      | Coffee      | Coffee      |
| 10:30-11:30 | Ricard      | de la Salle  | Parcet      | Ricard      | de la Salle |
| 11:30-13:30 | Lunch       | Lunch        | Lunch       | Lunch       | Lunch       |
| 13:30-14:30 | de la Salle | Parcet       | Ricard      | de la Salle | Parcet*     |
| 14:30-15:00 | Coffee      | Coffee       | Coffee      | Coffee      | Coffee*     |
| 15:00-15:40 | Coine       | Valette (1h) | Free        | Zadeh       | Free        |
| 15:50-16:30 | Krishnan    |              |             | Discussion  |             |
| 20:00-22:00 |             |              |             | Restaurant  |             |

The School Dinner on Thursday, November 21st, 2019, will take place at the Restaurant de la Charrette (11 Rue Jean Petit, 25000 Besançon) and will start at 20:00.

**Javier Parcet:** Singular integral methods for Fourier multipliers in group von Neumann algebras

**Éric Ricard:** Schur and Fourier multipliers

**Mikaël de la Salle:** Representation-theoretic methods for higher rank Fourier multipliers

\* Friday, Nov. 22nd, the afternoon lecture will be advanced to 13:00-14:00, and the coffee will be advanced to 14:00-14:30.

<sup>1</sup>supported by the French “Investissements d’Avenir” program, project ISITE-BFC (contract ANR-15-IDEX-03)

## Programme

### Monday, November 18, 2019

- 9:00-10:00 Lecture by **Javier Parcet**
- 10:00-10:30 Coffea and tea break
- 10:30-11:30 Lecture by **Éric Ricard**
- 11:30-13:30 Lunch in the student restaurant
- 13:30-14:30 Lecture by **Mikaël de la Salle**
- 14:30-15:00 Coffea and tea break
- 15:00-15:40 Talk by **Clément Coine**
- 15:50-16:30 Talk by **Arundhathi Krishnan**

### Tuesday, November 19, 2019

- 9:00-10:00 Lecture **Éric Ricard**
- 10:00-10:30 Coffea and tea break
- 10:30-11:30 Lecture **Mikaël de la Salle**
- 11:30-13:30 Lunch in the student restaurant
- 13:30-14:30 Lecture by **Javier Parcet**
- 14:30-15:00 Coffea and tea break
- 15:00-16:00 Talk by **Alain Valette**

### Wednesday, November 20, 2019

- 9:00-10:00 Lecture **Mikaël de la Salle**
- 10:00-10:30 Coffea and tea break
- 10:30-11:30 Lecture by **Javier Parcet**
- 11:30-13:30 Lunch in the student restaurant
- 13:30-14:30 Lecture **Éric Ricard**
- 14:30-15:00 Coffea and tea break

### Thursday, November 21, 2019

- 9:00-10:00 Lecture by **Javier Parcet**
- 10:00-10:30 Coffea and tea break
- 10:30-11:30 Lecture **Éric Ricard**
- 11:30-13:30 Lunch in the student restaurant
- 13:30-14:30 Lecture **Mikaël de la Salle**

**14:30-15:00** Coffea and tea break

**15:00-16:00** Talk by **Safoura Zadeh**

**20:00-22:00** Dinner in the Restaurant de la Charrette

## **Friday, November 22, 2019**

**9:00-10:00** Lecture **Éric Ricard**

**10:00-10:30** Coffea and tea break

**10:30-11:30** Lecture **Mikaël de la Salle**

**11:30-13:30** Lunch in the student restaurant

**13:00-14:00** Lecture by **Javier Parcet**

**14:00-14:30** Coffea and tea break

# Fourier multipliers in group von Neumann algebras

Javier Parcet – Éric Ricard – Mikael de la Salle

## Abstract

Let  $G$  be a locally compact (unimodular) group equipped with left regular representation  $\lambda: G \rightarrow \mathcal{U}(L_2(G))$  and corresponding group von Neumann algebra  $\mathcal{L}(G)$ . The Fourier multiplier associated to any bounded measurable function  $m: G \rightarrow \mathbf{C}$  is formally given by

$$f := \int_G \widehat{f}(g)\lambda(g)d\mu(g) \mapsto \int_G m(g)\widehat{f}(g)\lambda(g)d\mu(g) =: T_m f.$$

The analysis of Fourier multipliers in group algebras goes back to seminal work of Haagerup in the late 70's. In the last years, the  $L_p$ -boundedness of these maps and the corresponding class of Schur multipliers has been intensively investigated. This was essentially motivated for its connections in functional analysis, geometric group theory and harmonic analysis. Inspired by recent results for  $SL_n(\mathbf{R})$ , we shall present an overview of this line of research putting more emphasis in transference results between Fourier, Schur and Euclidean multipliers; rigidity theorems for multipliers over high rank semisimple Lie groups and lattices; and Mikhlin type regularity conditions which are sufficient for  $L_p$ -boundedness. All these points appear in our approach for  $SL_n(\mathbf{R})$ .

## Titles and abstracts for the afternoon talks

**Clément Coine:** Differentiability of operator functions in Schatten norms

Abstract: The differentiability of the functional calculus for selfadjoint matrices is linked with the study of Schur multipliers. If  $f : \mathbb{R} \rightarrow \mathbb{C}$  is continuously differentiable, then the functional calculus  $A \mapsto f(A)$  is differentiable and its derivative is given by a Schur multiplier in a basis for which  $A$  is diagonal.

In this talk, we discuss recent results on differentiability of operator functions in the infinite dimensional setting and for the Schatten norms. Let  $H$  be a separable Hilbert space and let, for any  $1 < p < \infty$ ,  $\mathcal{S}^p(H)$  be the Schatten class of order  $p$  on  $H$ . Let  $A$  be a (possibly unbounded) selfadjoint operator on  $H$  and let  $K = K^*$  be an element of  $\mathcal{S}^p(H)$ . Let  $f$  be a Lipschitz function on  $\mathbb{R}$ . We give higher order differentiability results for the function  $\varphi$  defined on  $\mathbb{R}$  by

$$\varphi : t \in \mathbb{R} \mapsto f(A + tK) - f(A) \in \mathcal{S}^p(H).$$

We will explain the assumptions we require on  $f$  and its derivatives to ensure the  $\mathcal{S}^p$ -differentiability of  $\varphi$  at the order  $n$  and we will give the representation of the derivatives of  $\varphi$ . As for application, we also derive a trace formula for operator Taylor remainders.

This is partly a joint work with C. Le Merdy, A. Skripka and F. Sukochev.

**Arundhathi Krishnan:** Markovianity and the Thompson monoid  $F^{++}$

Abstract: In the process of identifying a suitable distributional symmetry to describe Markovianity, it has been conjectured by C. Köstler that there is a certain correspondence between unilateral Markov shifts and representations of the Thompson monoid  $F^+$ . After having illustrated this correspondence in the context of tensor products of  $W^*$ -algebraic probability spaces, I will present the following two general results. A representation of the Thompson monoid  $F^+$  in the endomorphisms of a  $W^*$ -algebraic probability space yields a noncommutative Markov process (in the sense of Kümmerner). Conversely, such a representation is obtained from a noncommutative Markov process which is given as coupling to a so-called spreadable noncommutative Bernoulli shift.

**Alain Valette:** What is... weak amenability?

Abstract: A locally compact group  $G$  is weakly amenable if there exists a sequence of continuous compactly supported functions, converging to 1 uniformly on compacta of  $G$ , and bounded in the completely bounded norm on completely bounded multipliers of the Fourier algebra  $A(G)$ . Weak amenability was introduced in a famous paper by Cowling and Haagerup (1989). We will survey the developments of the subject, emphasizing the parallel with the close but inequivalent Haagerup property (or a-T-menability).

**Safoura Zadeh:**  $\ell^1$ -contractive maps on noncommutative  $L^p$ -spaces

Abstract: Let  $T: L^p(M) \rightarrow L^p(N)$  be a bounded operator between two noncommutative  $L^p$ -spaces,  $1 \leq p < \infty$ . We say that  $T$  is  $\ell^1$ -bounded if  $T \otimes I_{\ell^1}$  extends to a bounded map from  $L^p(M; \ell^1)$  into  $L^p(N; \ell^1)$ . In this talk, I discuss the relation between  $\ell^1$ -boundedness, regularity and positivity and how this concept can be applied to distinguish isometries  $T: L^2(M) \rightarrow L^2(N)$  with Yeadon type factorization. This is based on a joint work with Christian Le Merdy.