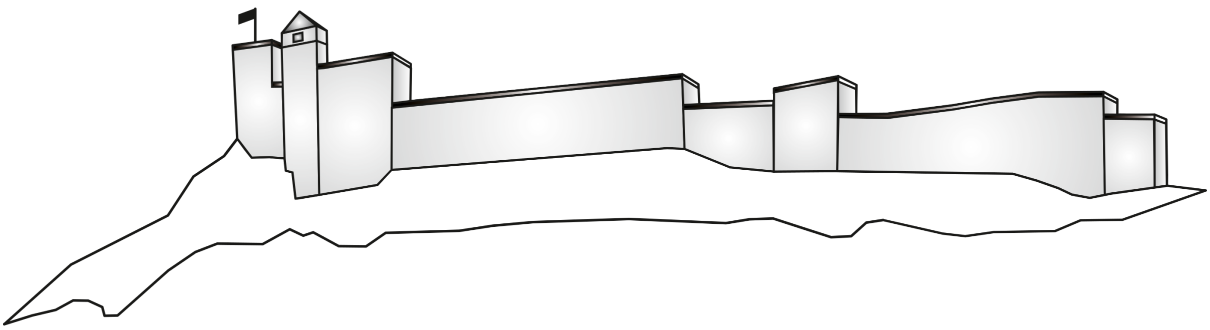
ECOLE DOCTORALE CARNOT - PASTEUR

UBFC UNIVERSITÉ BOURGOGNE FRANCHE-COMTÉ

The XXI meeting of the Carnot-Pasteur Doctoral School.

UFR Sciences et Techniques

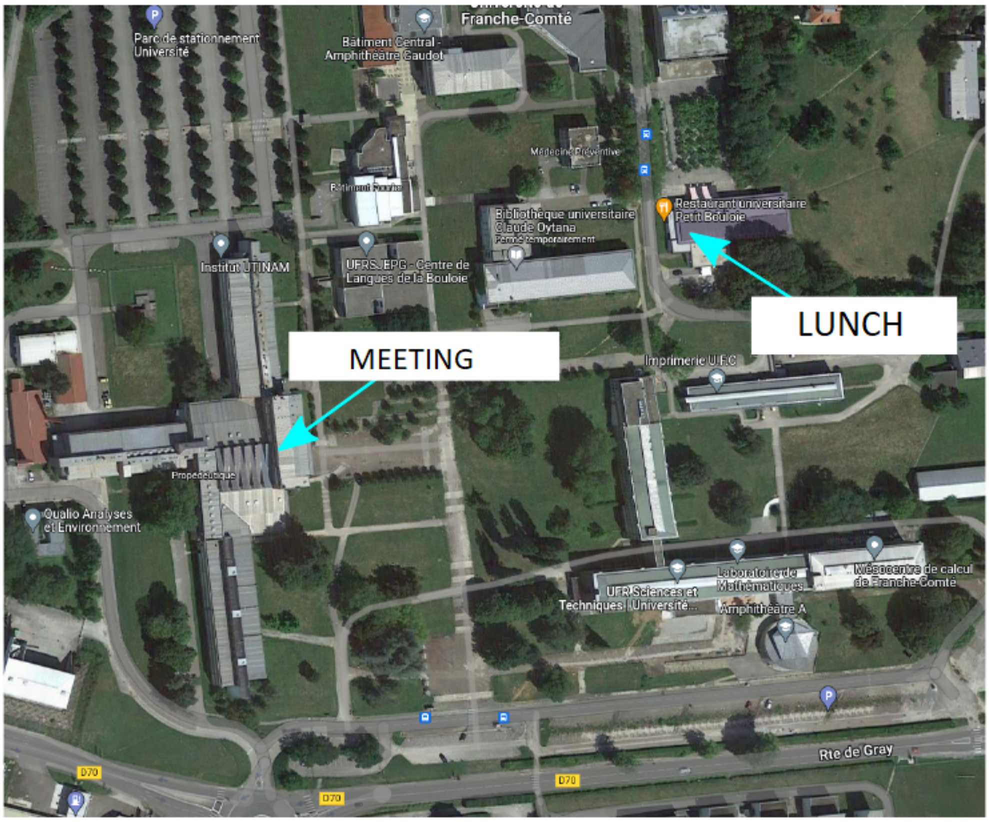
Université Bourgogne Franche-Comté



Besançon - June 24, 2022

Mathematics

ACCESS MAP.



Amphitheatre : Duffieux

Program.

8h30-9h: ______ REGISTRATION & INTRODUCTION.

9h-9h30: **Geometric Origin of the Tennis Racket Effect.** (Gabriela J. Gutierrez G, IMB).

9h30-10h: **High-dimensional consistency for model-assisted estimators in surveys.** (Mehdi Dagdoug, LMB)

10h-10h30: **Moduli space of super-instantons.** (Ouneïs Gloton, IMB)

10h30-11h: ______ COFFEE BREAK

11h-11h30: **Spatial statistical analysis of road crashes in Besançon.** (Cécile Spychala, LMB)

11h30-12h: **Topological equivalents pseudo-Anosov maps: A classification using Markov paritions.** (Inti Cruz Diaz,IMB)

12h-13h30: ______ LUNCH BREAK (R.U. Petit Bouloie)

13h30-13h40: **"Les enjeux environnementaux dans l'offre de formation doctorale."** (Julien Montillaud) **AMPHI CROISOT**.

13h40-14h10: **Non-divisible point on a two-parameter family of elliptic curves.** (Valentin Petit, LMB)

14h10-14h40: **Quantum Channel and Study of Semigroup of Positive Operators.** (Purbayan Chakraborty, LMB)

14h40-15h10: **Mathematical Properties of Continuous Ranked Probability Score Forecasting.** (Romain Pic, LMB)

15h10-15h40: ______ COFFEE BREAK

15h40-16h10: **Random matrix theory through the prism of statistical physics.** (Nicolas Babinet,IMB)

16h10-16h40: **On using optimal transport for an eigenvalue problem in electronic structure calculation.** (Maxime Dalery, LMB)

16h40-17h10: **Explicit reciprocity laws for formal Drinfeld modules.** (Marwa Ala Eddine, LMB)

17h10-17h30: ______ JURY DELIBERATION

17h30: ______ PRICE ANNOUNCEMENT

Geometric Origin of the Tennis Racket Effect

Gabriela J. Gutierrez G.,[1,*] P. Mardesic,[1] L. Van Damme, [2] D. Sugny[2]

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The tennis racket effect is a phenomenon that occurs in a free rotation of a rigid body. The experiment is as follows: the tennis racket is held by the handle and thrown in the air so that the handle makes a full turn, once the racket is caught, it is observed that the two faces of the racket have been exchanged. Therefore, the racket has performed an extra half-turn. This twist is called the tennis racket effect.

In a complex phase space, this effect originates from a pole of a 1-form defined over a Riemann surface revealing the geometric origin of the phenomenon. We obtain this result by analyzing a complex integral on this phase space. The tennis racket effect is not just an interesting phenomenon from a theoretical point of view, it also has applications in quantum control, where it is relevant because of its robustness to variations in initial conditions of the dynamics.

In this talk, I will explain the main idea behind all the elements mentioned before: the effect, the mathematical setting, the results, and the applications.

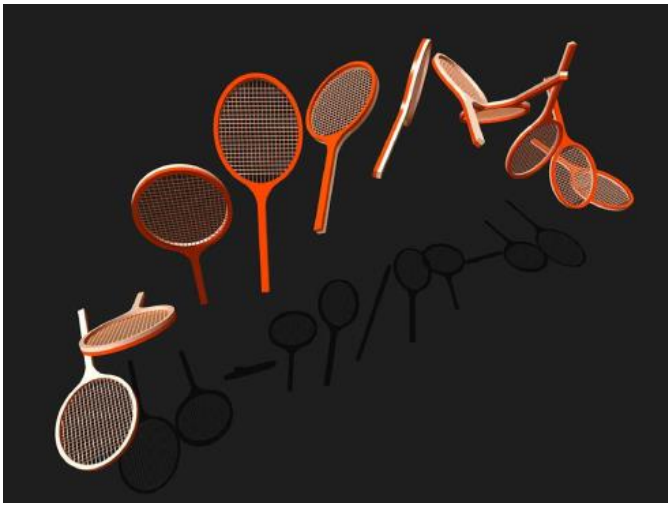

FIG. 1. The tennis racket effect.

[1] P. Mardesic, G.J. Gutierrez Guillen, L. Van Damme, D. Sugny, *Geometric origin of the tennis racket effect*, Physical Review Letters, 125 (2020)

[2] L. Van Damme, D. Leiner, P. Mardesic, S. J. Glaser, D. Sugny, *Linking the rotation of the rigid body to the Schrodinger equation: The quantum tennis racket effect and beyond*, Scientific reports, 7, pp.1-8 (2017)

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High-dimensional consistency for model-assisted estimators in surveys

Mehdi Dagdoug,[1,*] Camelia Goga,[1] and David Haziza[2]

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One of the main purposes of survey sampling is to estimate finite population parameters. Model-assisted estimators provide flexible tools for efficient estimation; these estimators can be used when additional information, called auxiliary variables, is available at the population level.
In most cases, the asymptotic properties of these estimators were established in a framework in which the population and the sample sizes were increasing to infinity, while the number of auxiliary variables was considered fixed. As such, this asymptotic framework considers that the number of auxiliary variables is negligible with respect to the sample size.
However, nowadays, it becomes more and more frequent for survey statisticians to face data with a large number of covariates, sometimes even of the order of the sample size. As such, in these scenarios, even for large population and sample sizes, the asymptotic results established in the previously mentioned framework cannot serve as good approximations of the behaviors of model-assisted estimators.

In this talk, after an introduction to the purposes and main concepts of survey sampling, I will consider a "high-dimensional" asymptotic framework in which the number of covariates is allowed to increase to infinity as well. The consistency of different model-assisted estimators will be established; a particular focus will be made on their convergence rates. A class of dimension-free model-assisted estimators will be exhibited. These theoretical results will also be illustrated with the results of a simulation study.

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Moduli space of super-instantons *(mathématiques)*

Ouneïs Gloton[1,*]

(Under the supervision of: Taro Kimura[1], Daniele Faenzi[1])

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The ***Yang-Mills equations*** arise in theoretical physics in the setting of quantum gauge theories, which form the basis of our modern understanding of particle physics. A class of solutions called ***instantons*** plays a role in the investigation of nonperturbative properties of quantum theory ([1] p.124). But they also turn out to be of great interest to mathematicians as S. K. Donaldson, in his groundbreaking work summarized in [2], used them to exhibit strong constraints on the topology of differentiable 4-manifolds.

The goal of this talk is to present in a first time, at a level accessible to other PhD students, the main object of Donaldson's construction: the ***moduli space of instantons***, at the interplay between differential geometry and algebraic geometry. In a second time, the proper subject of my PhD will be introduced: the mathematical generalization of this construction to gauge theories with a ***structure supergroup***, investigated in [3] from a theoretical physics perspective.

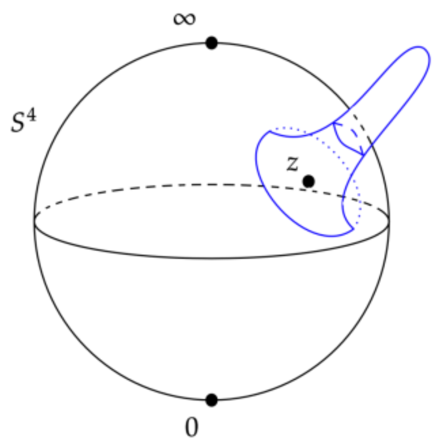

FIG. 1. A representation of an instanton
(credit: Visual representation of BPST Instanton field strength on 4-sphere by Tazerenix, used unmodified under CC BY-SA 4.0)

[1] Mariño, M. (2015). *Instantons and Large N: An Introduction to Non-Perturbative Methods in Quantum Field Theory*.
[2] Donaldson, S. K., & Kronheimer, P. B. (1990). *The geometry of four-manifolds*.
[3] Kimura, T. (2021). *Instanton Counting, Quantum Geometry and Algebra.*

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Spatial statistical analysis of road crashes in Besançon

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Road accidents being one of the main causes of death in the world are therefore the subject of serious concern. Statistical analyses of road accident data are major tools for law enforcement agencies who can act to prevent and predict these events. Depending on the main goal of the analysis, several methods can be undertaken. The focus of this study here is to identify the areas of the CAGB (urban community of Besançon) with high occurrence rates of road accidents. Moreover, the goal is also to identify the most dangerous related factors. Taking into account the environment characteristics of these accidents therefore seems essential. Road accident data of this study are georeferenced points and the main motivation is to understand how spatially these points are located. These events have been modelled by a spatial point process named log-Gaussian Cox process which enables to cross these georeferenced road crashes with environment characteristics in order to fulfill the study objectives [1].

[1] Taylor, B. M., Davies, T. M., Rowlingson, B. S., and Diggle, P. J. (2015). Bayesian inference and data augmentation schemes for spatial, spatiotemporal and multivariate log-gaussian cox processes in r. *Journal of Statistical Software*, 63(7).

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Topological equivalents pseudo-Anosov maps: A classification using Markov paritions

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Let S be an oriented surface and Hom$_+(S)$ be the group of oriented preserving homeomorphisms. Two maps $f, g \in$Hom(S) are *topologically equivalents* if there is another homeomorphism $h : S \to S$ such that, $f = h^{-1} \circ g \circ h$, the conjugacy class of f is denote $[f]$. In dynamical systems we are interested in understand such equivalent classes because quantities as entropy, number of periodic points or properties as mixing and transitivity are preserved (or not) for all elements in $[f]$.
Between the elements of Hom$_+(S)$ there is a family of particular maps, the so called . They are been study and classified from a topological approach by consider two maps equivalents if they are homotopic [**?**] , even more, there is an algorithm which determine if a homotopy class contain or not a pseudo-Anosov maps.
In a work in progress with Christian Bonatti we address the problem of classified the conjugacy classes of pseudo-Anosov maps. For us, to give a classification meas

1. Associate to every conjugacy class a finite family of objects, in such a way, the information let us to recover the conjugacy class. This information is called *geometric types*.
2. Determine what abstract geometrical types arise from pseudo-Anosov maps.
3. Give an algorithm to determine when two realizable geometric types generate conjugated pseudo Anosov maps.

The way to achieve this program is trough Markov partition, symbolic dynamics and a careful study of the invariant foliations of pseudo-Anosov map. In this talk i would like to present the objects of our research, try to point out why we find it interesting and give some hit of our methods.

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Non divisible point on two-parameters family of elliptic curves

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Laboratoire de Mathématiques de Besançon, Besançon, FRANCE

This work is focused on a family of two-parameters elliptic curves. The family considered is a generalization of the Washington's family. More precisely let $n \in \mathbb{N}^*$ and $t \in \mathbb{Z}_{\neq 0}$, we consider the elliptic curve defined over $\mathbb{Q}$ by

$$E\colon y^2 = x^3 + tx^2 - n^2(t + 3n^2)x + n^6.$$

The element $(0, n^3)$ is a point of $E(\mathbb{Q})$ of infinite order for all $n \in \mathbb{N}^*$, and $t \in \mathbb{Z}_{\neq 0}$. Under mild conditions, proved that the point $(0, n^3)$ is not divisible on E. Our work extends to this family of elliptic curve a previous study of Duquesne mainly stated for $n = 1$ and $t > 0$.

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Quantum Channel and Study of Semigroup of Positive Operators

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[2]*Indian Statistical Institute*

A quantum error basis is an abstract generalisation of Pauli matrices in higher dimensions which comes from a projective representation of a group G of n^2 elements. We can use this basis to study quantum maps and prove one to one correspondance between positive kernel K on $G \times G$ and completely positive(CP) maps which gives a general version of channel-state duality retrieving Choi-Jamiolkowski's result as a corollary. We will move further to characterise semigroups of CP maps and k-positive maps to prove a Schürman-Shcöenberg type correspondance between the semigroup and its generator.

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Mathematical Properties of Continuous Ranked Probability Score Forecasting

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The theoretical advances on the properties of scoring rules over the past decades have broaden the use of scoring rules in probabilistic forecasting. In meteorological forecasting, statistical postprocessing techniques are essential to improve the forecasts made by deterministic physical models. Numerous state-of-the-art statistical postprocessing techniques are based on distributional regression evaluated with the Continuous Ranked Probability Score (CRPS)[1]. However, theoretical properties of such minimization of the CRPS have mostly considered the unconditional framework (i.e. without covariables) and infinite sample sizes. We circumvent these limitations and study the rate of convergence in terms of CRPS of distributional regression methods We find the optimal minimax rate of convergence for a given class of distributions. Moreover, we show that the k nearest neighbor method and the kernel method for the distributional regression reach the optimal rate of convergence in dimension $d \geq 2$ and in any dimension, respectively.

Preprint : Mathematical Properties of Continuous Ranked Probability Score Forecasting, Pic et al. (https://arxiv.org/abs/2205.04360)

[1] James E. Matheson and Robert L. Winkler. Scoring rules for continuous probability distributions. Management Science, 22 (1976)

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Random matrix theory through the prism of statistical physics

Nicolas Babinet[1,*] and Taro Kimura[1]

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In this talk I will present some principles underlying the random matrix theory [1, 2]. I will in particular focus on the study of the eigenvalues of such matrices and their probability distribution which characterizes the corresponding model. The Gaussian case will be considered to illustrate those properties. One challenging problem of the random matrix theory is the understanding of the behavior of the eigenvalues when the matrix size is very large. The eigenvalue probability distribution in this asymptotic regime can be addressed throught the study of a Coulomb gas. This will lead us to another problem in statistical physics which can be described in the framework of random matrices [3]. Indeed, the study of a supermatrix model [4] coincide with a Coulomb gas made of particles with opposite charges. While the behavior in the thermodynamics limit is well understood for many years its behavior at finite temperature was unclear from the supermatrix perspective. I will therefore finish with a result we have got and with some new questions.

[1] Mehta, M.L., 1997. Random matrices and matrix models: The JNU lectures. Pramana - J Phys 48, 7–48.

[2] Eynard, B., Kimura, T., Ribault, S., 2018. Random matrices. arXiv:1510.04430

[3] D'adda, A., 1992. Comments on Supersymmetric Vector and Matrix Models. Class. Quantum Grav. 9, L21–L25.

[4] Alvarez-Gaumé, L., Mañes, J.L., 1991. Supermatrix models. Mod. Phys. Lett. A 06, 2039–2049.

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On using optimal transport for an eigenvalue problem in electronic structure calculation

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[1] *Laboratoire de Mathématiques de Besançon, France*

In electronic structure calculation, the computation of the ground state of a given molecular system is an important problem. Given a system with M nuclei characterized by their positions $X_1, \ldots, X_M$ in space and their electric charges $Z_1, \ldots, Z_M$, the ground state is the eigenvector with lowest energy of the Hamiltonian of the system. Here, we consider a simplified Hamiltonian for which analytical solutions are known.
We next present optimal transport theory and define the Wasserstein distance and study how it could be used to efficiently approximate the ground state.
A good hint is the comparison of the decay of Kolmogorov width between linear approximations and approximation using the Wasserstein metric. Finally we will discuss how we could take advantage of optimal transport for computing approximations of the ground state.

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Explicit reciprocity laws for formal Drinfeld modules

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We prove explicit reciprocity laws for a class of formal Drinfeld modules having stable reduction of height one, in the spirit of those existing in characteristic zero (cf. the work of Wiles [1]). We begin by defining the Kummer pairing in the language of formal Drinfeld modules defined over local fields of positive characteristic. We then prove explicit formulas for this pairing in terms of the logarithm of the formal Drinfeld module, a certain Coleman power series, torsion points and the trace. Our results extend the explicit formulas already proved by Anglès [2] for Carlitz modules, and by Bars and Longhi [3] for sign-normalized rank one Drinfeld modules.

[1] A. Wiles. Higher explicit reciprocity laws. Ann. of Math. (2), 107(2):235–254, 1978.
[2] B. Angl'es. On explicit reciprocity laws for the local Carlitz-Kummer symbols. J. Number Theory, 78(2):228–252, 1999
[3] F. Bars and I. Longhi. Coleman's power series and Wiles' reciprocity for rank 1 Drinfeld modules. J. Number Theory, 129(4):789–805, 2009

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