Boundary controlled port Hamiltonian systems

Y. Le Gorrec,

FEMTO-ST AS2M, Besançon, France
Joint work with : B. Maschke, H. Zwart, J. Villegas, H. Ramirez, A. Macchelli

February 18th, 2014

Outline

Recent technological progresses and physical knowledges allow to go toward the use of complex systems :

- Highly nonlinear.
- Involving numerous physical domains and possible heterogeneity.
- With distributed parameters or organized in network.

Recent technological progresses and physical knowledges allow to go toward the use of complex systems :

- Highly nonlinear.
- Involving numerous physical domains and possible heterogeneity.
- With distributed parameters or organized in network.

New issue for system control theory

Modelling step is important $\longrightarrow$ the physical properties can be advantageously used for analysis, control or simulation purposes

Example 1 : Ionic Polymer Metal Composite

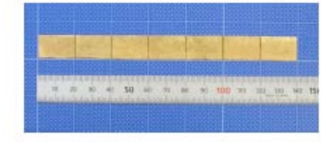
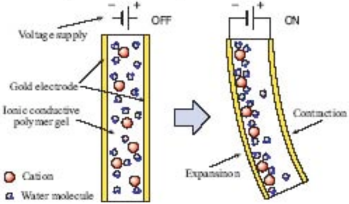
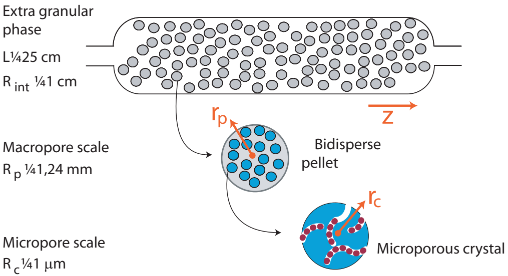


Figure 3. Beam-shaped IPMC actuator



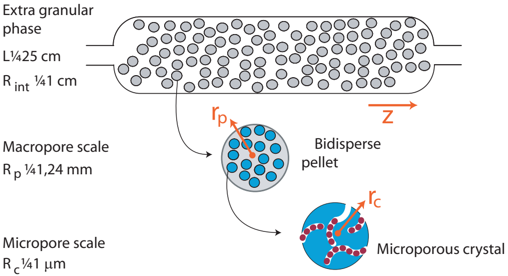
- Electromechanical system.
- 3 scales : Polymer-electrode interface, diffusion in the polymer, beam bending.

Example 2 : Adsorption process



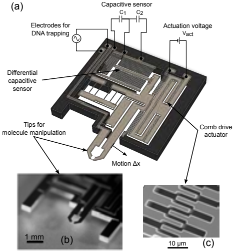
- Multiscale heterogeneous system.
- Dynamic behavior driven by irreversible thermodynamic laws

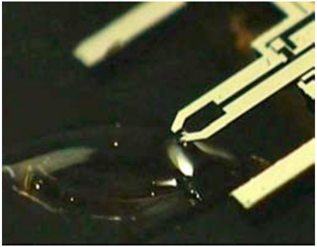
Example 2 : Adsorption process

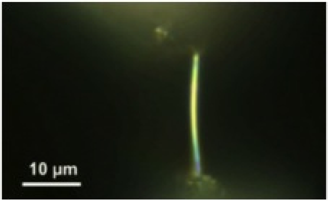


- Multiscale heterogeneous system.
- Considered phenomena :
  - Fluid scale : convection, dispersion.
  - Pellet scale : diffusion (Stephan-Maxwell).
  - Microscopic scale : Knudsen law.

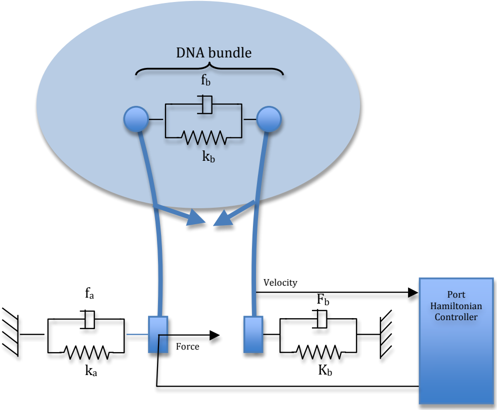
Example 3 : Nanotweezer for DNA manipulation







Example 3 : Nanotweezer for DNA manipulation



Port Hamiltonian framework

Port Hamiltonian systems

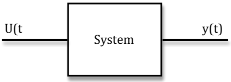
Class of non linear dynamic systems derived from an extension to open physical systems (1992) of Hamiltonian and Gradient systems. This class has been generalized (2001) to distributed parameter systems.

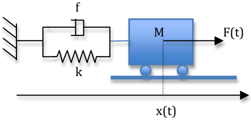
$$x(t): \begin{cases} \dot{x} = (J(x) - R(x)) \frac{\partial H(x)}{\partial x} + B(x)u \\ y = B(x)^T \frac{\partial H(x)}{\partial x} \end{cases} \qquad x(t,z): \begin{cases} \dot{x} = (\mathcal{J}(x) - \mathcal{R}(x)) \frac{\delta \mathcal{H}(x)}{\delta x} \\ \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \frac{\delta \mathcal{H}(x)}{\delta x}\Big|_\partial \end{cases}$$

- Central role of the energy.
- Additional information coming from the geometric structure.
- Multi-physic framework.

A simple example ...

Let consider the mass spring damper system :





From the second Newton's law :

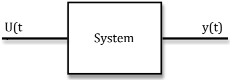
$$M\ddot{x} = -kx - f\dot{x} + F$$

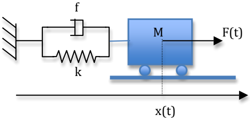
which is usually treated using the canonical state space representation :

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{k}{M} & -f \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} F$$

A simple example ...

Let consider the mass spring damper system :





From the second Newton's law :

$$M\ddot{x} = -kx - f\dot{x} + F$$

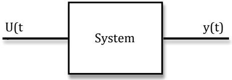
An alternative representation consist in choosing the energy variables (extensives variables) as state variables *i.e* $(x, p = M\dot{x})$

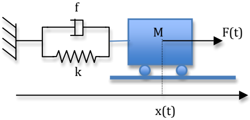
$$\begin{pmatrix} \dot{x} \\ \dot{p} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & -f \end{pmatrix}}_{J-R} \underbrace{\begin{pmatrix} kx \\ \dot{x} \end{pmatrix}}_{\partial_x H} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{B} F$$

with $H(x,p) = kx^2 + \frac{1}{M}p^2$

A simple example ...

Let consider the mass spring damper system :





From the second Newton's law :

$$M\ddot{x} = -kx - f\dot{x} + F$$

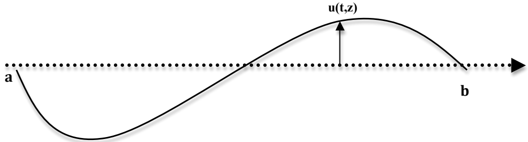
Defining y s.t. :

$$\begin{cases} \begin{pmatrix} \dot{x} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -f \end{pmatrix} \begin{pmatrix} \partial_x H(x,p) \\ \partial_p H(x,p) \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} F \\ y = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \partial_x H(x,p) \\ \partial_p H(x,p) \end{pmatrix} \end{cases}$$

$$\frac{dH}{dt} = \frac{\partial H}{\partial x}^T \frac{dx}{dt} = \frac{\partial H}{\partial x}^T (J - R) \frac{\partial H}{\partial x} + \frac{\partial H}{\partial x}^T Bu \leq y^T u$$

A second example ....

Vibrating string :



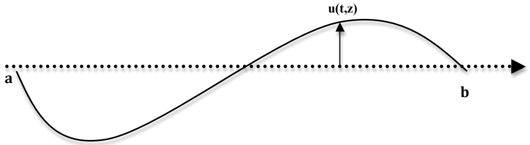
The classical modelling is based on the wave equation : Newton's law + Hooke's law (restoring force proportional to the deformation)

$$\frac{\partial^2 u(z,t)}{\partial t^2} = \frac{1}{\mu(z)} \frac{\partial}{\partial z} \left(T(z) \frac{\partial u(z,t)}{\partial z} \right)$$

The structure of the model is not apparent. How to choose the boundary conditions ? ? ?

A second example ....

Vibrating string :



The classical modelling is based on the wave equation : Newton's law + Hooke's law (restoring force proportional to the deformation)

$$\frac{\partial^2 u(z,t)}{\partial t^2} = \frac{1}{\mu(z)} \frac{\partial}{\partial z} \left(T(z) \frac{\partial u(z,t)}{\partial z} \right)$$

The structure of the model is not apparent. How to choose the boundary conditions ? ? ?

Usually : $x = \begin{bmatrix} u \\ \dot{u} \end{bmatrix} \rightarrow \begin{bmatrix} u \\ \dot{u} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{1}{\mu(z)} \frac{\partial}{\partial z} \left(T(z) \frac{\partial .}{\partial z} \right) & 0 \end{bmatrix} \begin{bmatrix} u \\ \dot{u} \end{bmatrix}$ first order diff equation in time

A second example ...

Let choose as state variables the energy variables :

- the strain $\varepsilon = \frac{\partial u(z,t)}{\partial z}$
- the elastic momentum $p = \mu(z) v(z,t)$

The **total energy** is given by : $H(\varepsilon, p) = U(\varepsilon) + K(p)$

- $U(\varepsilon)$ is the **elastic potential energy** :

$$U(\varepsilon) = \int_a^b \frac{1}{2} T(z) \left(\frac{\partial u(z,t)}{\partial z} \right)^2 = \int_a^b \frac{1}{2} T\varepsilon(z,t)^2$$

  where $T(z)$ denotes the elastic modulus.

- $K(v)$ is the **kinetic energy** :

$$K(p) = \int_a^b \frac{1}{2} \mu(z) v(z,t)^2 = \int_a^b \frac{1}{2} \frac{1}{\mu(z)} p^2(z,t)$$

  where $\mu(z)$ denotes the string mass.

A second example ...

From the conservation laws :

$$\frac{\partial}{\partial t}\left(\begin{array}{c}\varepsilon\\ p\end{array}\right)+\frac{\partial}{\partial z}\left(\begin{array}{c}v\\ \sigma\end{array}\right)=0$$

where $v(z,t)$ is the velocity and $\sigma(z,t)=T(z)\varepsilon(z,t)$ the stress.
The vector of fluxes β may be expressed in term of the generating forces :

$$\left(\begin{array}{c}v\\ \sigma\end{array}\right)=\underbrace{\left(\begin{array}{cc}0&1\\ 1&0\end{array}\right)}_{\substack{\text{canonical}\\ \text{inerdomain coupling}}}\underbrace{\left(\begin{array}{c}\frac{\delta H}{\delta \varepsilon}\\ \frac{\delta H}{\delta p}\end{array}\right)}_{\substack{\text{generating}\\ \text{forces}}}$$

Consequently

$$\frac{\partial}{\partial t}\left(\begin{array}{c}\varepsilon\\ p\end{array}\right)=-\frac{\partial}{\partial z}\left(\begin{array}{cc}0&1\\ 1&0\end{array}\right)\left(\begin{array}{c}\frac{\delta H}{\delta \varepsilon}\\ \frac{\delta H}{\delta p}\end{array}\right)$$

A second example ...

From the conservation laws :

$$\frac{\partial}{\partial t}\begin{pmatrix}\varepsilon \\ p\end{pmatrix} + \frac{\partial}{\partial z}\begin{pmatrix}v \\ \sigma\end{pmatrix} = 0$$

where $v(z,t)$ is the velocity and $\sigma(z,t) = T(z)\varepsilon(z,t)$ the stress.
The vector of fluxes β may be expressed in term of the generating forces :

$$\begin{pmatrix}v \\ \sigma\end{pmatrix} = \underbrace{\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}}_{\substack{\text{canonical} \\ \text{inerdomain coupling}}} \underbrace{\begin{pmatrix}\frac{\delta H}{\delta \varepsilon} \\ \frac{\delta H}{\delta p}\end{pmatrix}}_{\substack{\text{generating} \\ \text{forces}}}$$

PDEs :

$$\frac{\partial}{\partial t}\begin{pmatrix}\varepsilon \\ p\end{pmatrix} = \begin{pmatrix}0 & -\frac{\partial}{\partial z} \\ -\frac{\partial}{\partial z} & 0\end{pmatrix}\begin{pmatrix}\frac{\delta H}{\delta \varepsilon} \\ \frac{\delta H}{\delta p}\end{pmatrix} \Leftrightarrow \frac{\partial^2 u(z,t)}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 u(z,t)}{\partial z^2} \text{ if } c = cte$$

+BC

Port Hamiltonian structure

Underlying structure :

$$\underbrace{\frac{\partial}{\partial t}\begin{pmatrix}\varepsilon\\ p\end{pmatrix}}_{f} = \underbrace{\begin{pmatrix}0 & -\frac{\partial}{\partial z}\\ -\frac{\partial}{\partial z} & 0\end{pmatrix}}_{\substack{\mathcal{J} = \text{matrix}\\ \text{differential operator}}} \underbrace{\begin{pmatrix}T(z) & 0\\ 0 & \frac{1}{\mu(z)}\end{pmatrix}\begin{pmatrix}\varepsilon\\ p\end{pmatrix}}_{\substack{e = \text{driving}\\ \text{force}}}$$

Hamiltonian operator $\mathcal{J}$ is **skew-symmetric only for function with compact domain strictly** in Z :

$$\int_a^b \begin{pmatrix} e_1 & e_2 \end{pmatrix} \mathcal{J}\begin{pmatrix} e_1'\\ e_2'\end{pmatrix} + \begin{pmatrix} e_1' & e_2' \end{pmatrix}\mathcal{J}\begin{pmatrix} e_1\\ e_2\end{pmatrix} = -\left[e_1 e_2' + e_2 e_1'\right]_a^b$$

Power balance equation :

$$\begin{aligned}\frac{d}{dt}H(\varepsilon,p) &= \int_a^b \left(\frac{\delta\mathcal{H}}{\delta\varepsilon}\frac{\partial\varepsilon}{\partial t} + \frac{\delta\mathcal{H}}{\delta p}\frac{\partial p}{\partial t}\right) dz\\ &= -\int_a^b \left(\frac{\delta\mathcal{H}}{\delta\varepsilon}\frac{\partial}{\partial z}\frac{\delta\mathcal{H}}{\delta p} + \frac{\delta\mathcal{H}}{\delta p}\frac{\partial}{\partial z}\frac{\delta H}{\delta\varepsilon}\right) dz = -\left[\frac{\delta\mathcal{H}}{\delta\varepsilon}\frac{\delta\mathcal{H}}{\delta p}\right]_a^b\end{aligned}$$

If driving forces are zero at the boundary, the total energy is conserved, else there is a **flow of power at the boundary**. Define two **port boundary variables** as follows :

$$\begin{pmatrix} f_\partial\\ e_\partial\end{pmatrix} = \begin{pmatrix}\frac{\delta H}{\delta\varepsilon}\\ \frac{\delta H}{\delta p}\end{pmatrix}\Big|_{a,b}$$

Port Hamiltonian structure

The linear space $\mathcal{D} \ni (f_1, f_2, e_1, e_2, f_\partial, e_\partial)$

- $\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{\partial}{\partial z} \\ -\frac{\partial}{\partial z} & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$
- $\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} |_{a,b}$

defines a **Dirac structure :**$\mathcal{D} = \mathcal{D}^\perp$ with respect to the pairing :

$$\int_a^b e_1 f_1 dz + \int_a^b e_2 f_2 dz - f_\partial^T e_\partial$$

Port Hamiltonian system

$$\left(\frac{\partial}{\partial t}\alpha, \frac{\delta H}{\delta \alpha}, f_\partial, e_\partial \right) \in \mathcal{D}$$

Port Hamiltonian structure

The linear space $\mathcal{D} \ni (f_1, f_2, e_1, e_2, f_\partial, e_\partial)$

- $\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{\partial}{\partial z} \\ -\frac{\partial}{\partial z} & 0 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$
- $\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} |_{a,b}$

defines a **Dirac structure :**$\mathcal{D} = \mathcal{D}^\perp$ with respect to the pairing :

$$\int_a^b e_1 f_1 dz + \int_a^b e_2 f_2 dz - f_\partial^T e_\partial$$

Port Hamiltonian system

$$\left(\frac{\partial}{\partial t}\alpha, \frac{\delta H}{\delta \alpha}, f_\partial, e_\partial \right) \in \mathcal{D}$$

$$\frac{dH}{dt} = f_\partial^T e_\partial$$

Outline

1. Introduction

2. Boundary controlled port Hamiltonian systems

3. Asymptotic stability

4. Exponential stability

5. Conclusion and future work

Boundary controlled port Hamiltonian systems

Considered class of PDEs (1D)

$$\frac{\partial x(t,z)}{\partial t} = \mathcal{J}\, \delta_x \mathcal{H}(x(t,z)), \quad \text{with } \mathcal{J} \text{ skew sym. diff. operator}$$

Boundary controlled port Hamiltonian systems

Considered class of PDEs (1D)

$$\frac{\partial x}{\partial t} = P_1 \frac{\partial}{\partial z}(\mathcal{L}(z)x)(t,z)) + (P_0 - G_0)\mathcal{L}(z)x(t,z)$$

$P_1 = P_1^\top$, $P_0 = -P_0^\top$, $G_0 \geq 0$, $x \in \mathbb{R}^n$, $z \in (a,b)$, $\mathcal{L}(z) = \mathcal{L}(z)^\top > 0$. State space $X = L_2(a,b;\mathbb{R}^n)$ with $\langle x_1, x_2 \rangle_{\mathcal{L}} = \langle x_1, \mathcal{L}x_2 \rangle$ and the norm $\|x_1\|^2_{\mathcal{L}} = \langle x_1, x_1 \rangle_{\mathcal{L}}$.

Boundary controlled port Hamiltonian systems

Considered class of PDEs (1D)

$$\frac{\partial x}{\partial t} = P_1 \frac{\partial}{\partial z}\left(\mathcal{L}(z)x)(t,z)\right) + (P_0 - G_0)\mathcal{L}(z)x(t,z)$$

$P_1 = P_1^\top$, $P_0 = -P_0^\top$, $G_0 \geq 0$, $x \in \mathbb{R}^n$, $z \in (a,b)$, $\mathcal{L}(z) = \mathcal{L}(z)^\top > 0$. State space $X = L_2(a,b;\mathbb{R}^n)$ with $\langle x_1, x_2\rangle_{\mathcal{L}} = \langle x_1, \mathcal{L}x_2\rangle$ and the norm $\|x_1\|^2_{\mathcal{L}} = \langle x_1, x_1\rangle_{\mathcal{L}}$.

The norm $\|\cdot\|^2_{\mathcal{L}}$ is equivalent to the energy of the system

Applications

- Mechanical systems, magneto-electro-mechanical, chemical, etc...
- Some beam and wave equations, Maxwell equations, transmission lines, vibrating strings, Saint-Venant equations, ...
- But also by using appropriate extension + closure relations : heat transmission, diffusion systems, tubular reactors, etc...

Boundary controlled port Hamiltonian systems

Considered class of PDEs (1D)

$$\frac{\partial x}{\partial t} = P_1 \frac{\partial}{\partial z}(\mathcal{L}(z)x)(t,z)) + (P_0 - G_0)\mathcal{L}(z)x(t,z)$$

$P_1 = P_1^\top$, $P_0 = -P_0^\top$, $G_0 \geq 0$, $x \in \mathbb{R}^n$, $z \in (a,b)$, $\mathcal{L}(z) = \mathcal{L}(z)^\top > 0$. State space $X = L_2(a,b;\mathbb{R}^n)$ with $\langle x_1, x_2\rangle_{\mathcal{L}} = \langle x_1, \mathcal{L}x_2\rangle$ and the norm $\|x_1\|^2_{\mathcal{L}} = \langle x_1, x_1\rangle_{\mathcal{L}}$.

Boundary port variables

Let $\mathcal{L}x \in H^1(a,b;\mathbb{R}^n)$. Then the boundary port variables are the vectors $e_{\partial,\mathcal{L}x}, f_{\partial,\mathcal{L}x} \in \mathbb{R}^n$,

$$\begin{bmatrix} f_{\partial,\mathcal{L}x} \\ e_{\partial,\mathcal{L}x} \end{bmatrix} = U \frac{1}{\sqrt{2}} \begin{bmatrix} P_1 & -P_1 \\ I & I \end{bmatrix} \begin{bmatrix} (\mathcal{L}x)(b) \\ (\mathcal{L}x)(a) \end{bmatrix} = R \begin{bmatrix} (\mathcal{L}x)(b) \\ (\mathcal{L}x)(a) \end{bmatrix}.$$

Where

$$U^T \Sigma U = \Sigma, \quad \Sigma = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \qquad \Sigma \in M_{2n}(\mathbb{R})$$

Boundary controlled port Hamiltonian systems

Boundary controlled port Hamiltonian systems [Le Gorrec et al., 2005]

Let W be a $n \times 2n$ real matrix. If W has full rank and satisfies $W \Sigma W^\top \geq 0$, then the system $\frac{\partial x}{\partial t} = P_1 \frac{\partial}{\partial z} (\mathcal{L}(z)x)(t,z)) + (P_0 - G_0)\mathcal{L}(z)x(t,z)$with input

$$u(t) = W \begin{bmatrix} f_{\partial,\mathcal{L}x}(t) \\ e_{\partial,\mathcal{L}x}(t) \end{bmatrix}$$

is a BCS on X. The operator $\mathcal{A}x = P_1(\partial/\partial z)(\mathcal{L}x) + (P_0 - G_0)\mathcal{L}x$ with domain

$$D(\mathcal{A}) = \left\{ \mathcal{L}x \in H^1(a,b;\mathbb{R}^n) \;\middle|\; \begin{bmatrix} f_{\partial,\mathcal{L}x}(t) \\ e_{\partial,\mathcal{L}x}(t) \end{bmatrix} \in \ker W \right\}$$

generates a contraction semigroup on X.

Boundary controlled port Hamiltonian systems

Boundary controlled port Hamiltonian systems [Le Gorrec et al., 2005]

Let $\tilde{W}$ be a full rank matrix of size $n \times 2n$ with $\begin{bmatrix} W \\ \tilde{W} \end{bmatrix}$ invertible and let $P_{W,\tilde{W}}$ be given by

$$P_{W,\tilde{W}} = \left(\begin{bmatrix} W \\ \tilde{W} \end{bmatrix} \Sigma \begin{bmatrix} W \\ \tilde{W} \end{bmatrix}^\top \right)^{-1} = \begin{bmatrix} W\Sigma W^\top & W\Sigma \tilde{W}^\top \\ \tilde{W}\Sigma W^\top & \tilde{W}\Sigma \tilde{W}^\top \end{bmatrix}^{-1}.$$

Define the output of the system as the linear mapping $\mathcal{C} : \mathcal{L}^{-1} H^1(a, b; \mathbb{R}^n) \to \mathbb{R}^n$,

$$y = \mathcal{C}x(t) := \tilde{W} \begin{bmatrix} f_{\partial,\mathcal{L}x}(t) \\ e_{\partial,\mathcal{L}x}(t) \end{bmatrix}.$$

Then for $u \in C^2(0, \infty; \mathbb{R}^k)$, $\mathcal{L}x(0) \in H^1(a, b; \mathbb{R}^n)$, and $u(0) = W \begin{bmatrix} f_{\partial,\mathcal{L}x}(0) \\ e_{\partial,\mathcal{L}x}(0) \end{bmatrix}$ the following balance equation is satisfied :

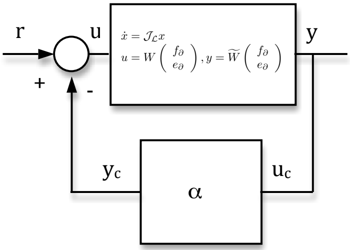
$$\frac{1}{2}\frac{d}{dt}\|x(t)\|^2_{\mathcal{L}} = \frac{1}{2} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix}^\top P_{W,\tilde{W}} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} - \langle G_0 \mathcal{L}x(t), \mathcal{L}x(t) \rangle \leq \frac{1}{2} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix}^\top P_{W,\tilde{W}} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix}.$$

Closed loop control

Impedance passive case

As it has been pointed out in [Villegas, 2007], if the matrices W and $\tilde{W}$ are selected such that $P_{W,\tilde{W}} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} = \Sigma$, then the BCS fulfills

$$\frac{1}{2}\frac{d}{dt}\|x(t)\|^2_{\mathcal{L}} \leq u^\top(t)y(t).$$



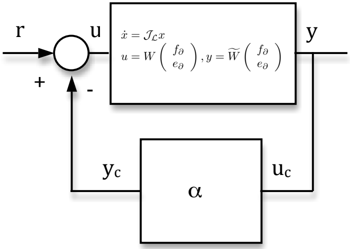
$$\begin{cases} \dot{x} = \mathcal{J}_{\mathcal{L}} x \\ r = \left(W + \alpha \widetilde{W}\right) \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} \\ y = \widetilde{W} \begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} \end{cases}$$

Closed loop control

Impedance passive case

As it has been pointed out in [Villegas, 2007], if the matrices W and $\tilde{W}$ are selected such that $P_{W,\tilde{W}} = \left[\begin{smallmatrix} 0 & I \\ I & 0 \end{smallmatrix}\right] = \Sigma$, then the BCS fulfills

$$\frac{1}{2}\frac{d}{dt}\|x(t)\|^2_{\mathcal{L}} \leq u^\top(t)y(t).$$



Static controller

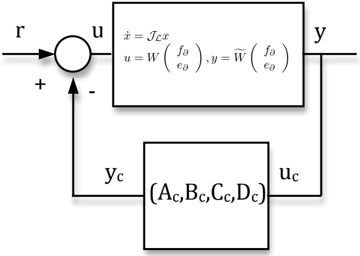
- Asymptotic stability : $\alpha > 0$+(compacness condition)
- Exponential stability : sufficient condition on the choice of the closed loop BC

Closed loop control

Impedance passive case

As it has been pointed out in [Villegas, 2007], if the matrices W and $\tilde{W}$ are selected such that $P_{W,\tilde{W}} = \left[\begin{smallmatrix} 0 & I \\ I & 0 \end{smallmatrix}\right] = \Sigma$, then the BCS fulfills

$$\frac{1}{2}\frac{d}{dt}\|x(t)\|^2_{\mathcal{L}} \leq u^\top(t)y(t).$$



$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} \mathcal{J}\mathcal{L} & 0 \\ B_c\mathcal{C} & A_c \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}$$

with $D(\mathcal{A}_e) = \left\{ \begin{bmatrix} x \\ v \end{bmatrix} \in \begin{bmatrix} x \\ v \end{bmatrix} \middle| \mathcal{L}x \in H^N(a,b;\mathbb{R}^n), \begin{bmatrix} f_{\partial,\mathcal{L}x} \\ e_{\partial,\mathcal{L}x} \\ v \end{bmatrix} \in \ker \tilde{W}_D \right\}$,

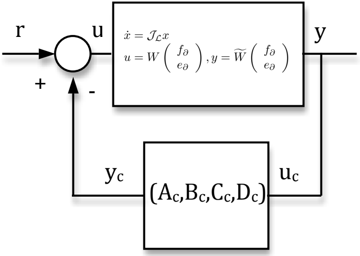
where $\tilde{W}_D = \left[(W + D_c\tilde{W} \quad C_c) \right]$

Closed loop control

Impedance passive case

As it has been pointed out in [Villegas, 2007], if the matrices W and $\tilde{W}$ are selected such that $P_{W,\tilde{W}} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} = \Sigma$, then the BCS fulfills

$$\frac{1}{2}\frac{d}{dt}\|x(t)\|^2_{\mathcal{L}} \leq u^\top(t)y(t).$$



Dynamic controller

- Asymptotic stability : $G(s)$ strictly positive real +(compactness condition)

Outline

Dynamic boundary feedback

Consider a strictly passive linear finite dimensional system

$$\dot{v} = A_c v + B_c u_c, \qquad y_c = C_c v + D_c u_c.$$

with storage function $E_c(t) = \frac{1}{2}\langle v(t) Q_c v(t)\rangle_{\mathbb{R}^m}$, $Q_c = Q_c^\top > 0 \in \mathbb{R}^m \times \mathbb{R}^m$.

Theorem [Villegas, 2007]

Let the open-loop BCS satisfy $\frac{1}{2}\frac{d}{dt}\|x(t)\|^2_{\mathcal{L}} = u(t)y(t)$. Consider a LTI strictly passive finite dimensional system with storage function $E_c(t) = \frac{1}{2}\langle v(t), Q_c v(t)\rangle_{\mathbb{R}^m}$. Then the power preserving feedback interconnection

$$u = r - y_c, \qquad y = u_c,$$

with $r \in \mathbb{R}^n$ the new input of the system is a BCS on the extended state space $\tilde{x} \in \tilde{X} = X \times V$ with inner product $\langle \tilde{x}_1, \tilde{x}_2\rangle_{\tilde{X}} = \langle x_1, x_2\rangle_{\mathcal{L}} + \langle v_1, Q_c v_2\rangle_V$. Furthermore, the operator $\mathcal{A}_e$ defined by

$$\mathcal{A}_e \tilde{x} = \begin{bmatrix} \mathcal{J}\mathcal{L} & 0 \\ B_c\mathcal{C} & A_c \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}, \quad D(\mathcal{A}_e) = \left\{ \begin{bmatrix} x \\ v \end{bmatrix} \in \begin{bmatrix} X \\ V \end{bmatrix} \middle| \mathcal{L}x \in H^N(a, b; \mathbb{R}^n), \begin{bmatrix} f_{\partial,\mathcal{L}x} \\ e_{\partial,\mathcal{L}x} \\ v \end{bmatrix} \in \ker \tilde{W}_D \right\}$$

where

$$\tilde{W}_D = \left[(W + D_c \tilde{W} \quad C_c) \right]$$

generates a contraction semigroup on $\tilde{X}$.

Asymptotic stability

Finite dimensional port Hamiltonian controller

$$\dot{v} = (J_c - R_c)Q_c v + B_c u_c, \quad y_c = B_c^\top Q_c v, \quad E_c(t) = \frac{1}{2} v(t)^\top Q_c v(t)$$

where we assume that $Q_c = Q_c^\top > 0$, $J_c = -J_c^\top$, $R_c = R_c^\top \geq 0$ and B_c are real constant matrices of proper dimensions. Furthermore, the controller is assumed to be exponentially stable, i.e., $A_c := (J_c - R_c)Q_c$ is Hurwitz.

Theorem

Consider the above controller connected to the impedance passive system through $u = r - y_c, u_c = y$. Then the operator $\mathcal{A}_e$ described in the previous theorem has compact resolvant.

Theorem

Consider the feedback system $u = r - y_c, u_c = y$ where the controller is chosen satisfying the condition above. Then the closed loop system such that $r = 0$ is globally asymptotically stable.

Sketch of proof

- Let first consider that $\omega(0) \in D(\mathcal{A}_e)$. By the aforementioned Theorem [Villegas, 2007], $\mathcal{A}_e$ generates a contraction semigroup.
- Let now consider the energy as Lyapunov function $E_c(t) = \frac{1}{2}\langle \omega(t), \omega(t)\rangle_{\tilde{X}}$. Since $\omega(0) \in D(\mathcal{A}_e)$ and :

$$\frac{dE_c(t)}{dt} = \langle \dot{\omega}(t), \omega(t)\rangle_{\tilde{X}} = \langle \mathcal{A}_e \omega(t), \omega(t)\rangle_{\tilde{X}} = -v^T Q_d v \quad (1)$$

  where $Q_d > 0$. Since $(\lambda I - \mathcal{A}_e)^{-1}$ is compact and the semigroup is a contraction it follows from LaSalle's invariance principle that all solutions asymptotically tend to the maximal invariant set $\mathcal{O}_c = \left\{\tilde{x} \in \tilde{X} | \dot{E}_c = 0\right\}$.
- Let $\mathcal{E}$ be the largest invariant subset of $\mathcal{O}_c$. We can prove that $\mathcal{E} = \{0\}$. From $\dot{E}_c(t) = 0$ and (1) we have $v(t) = 0$ and then $\dot{v}(t) = 0$. Let $\eta < n$ be the rank of $\ker(B_c)$. Form the controller structure $y_c = 0$ and $n - \eta > 0$ components of u_c equal 0. It follows that $\mathcal{O}_c$ reduces to the solution of a first order PDE of dimension n with $2n - \eta$ boundary variables set to zero. It follows from Holmgren's Theorem that $\tilde{x}(t) = 0$, hence the asymptotic stability. The same hold for $\omega(0) \in \tilde{X}$ by using denseness argument John78.

Energy shaping

From the power preserving interconnexion :

$$\tilde{E}(x, v) = E(x) + E_c(v)$$

We are looking for Casimirs on the form :

$$C(x, v) = v + F(x)$$

then

$$v + F(x) = \kappa$$

And

$$\tilde{E}(x, v) = \tilde{E}(x) = E(x) + E_c(-F(x) + \kappa)$$

It remains to choose E_c s.t.

$$\frac{\partial \tilde{E}}{\partial x}(x^*) = 0$$

Casimirs

Let consider the structural invariants of the closed loop system *i.e.* Casimirs, of the form :

$$C(x(t), v(t)) = \Gamma^\top v(t) + \int_a^b \Psi^\top(z) x(t,z) dz \qquad (2)$$

with $\Gamma \in \mathbb{R}^m$, $\Psi(z) \in \mathbb{R}^n$ and $\Psi^\top(z)x(t,z) \in H^1(a,b;\mathbb{R}^n)$.

Casimirs

Consider the previously defined BCS with $r = 0$, where the controller is a dissipative port Hamiltonian controller. Then (2) is a Casimir function for the extended system if :

$$P_1 \frac{\partial}{\partial z}\Psi(z) + (P_0 + G_0)\Psi(z) = 0, \qquad (3)$$

$$(J_c + R_c)\Gamma + B_c \tilde{W} R \begin{bmatrix} \Psi(b) \\ \Psi(a) \end{bmatrix} = 0, \qquad (4)$$

$$B_c^\top \Gamma + W R \begin{bmatrix} \Psi(b) \\ \Psi(a) \end{bmatrix} = 0. \qquad (5)$$

Outline

1. Introduction

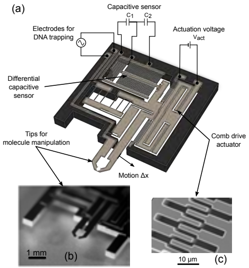
2. Boundary controlled port Hamiltonian systems

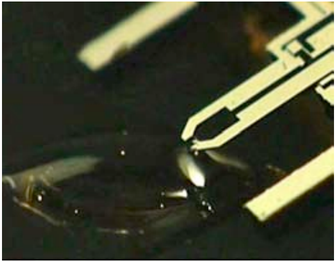
3. Asymptotic stability

4. Exponential stability

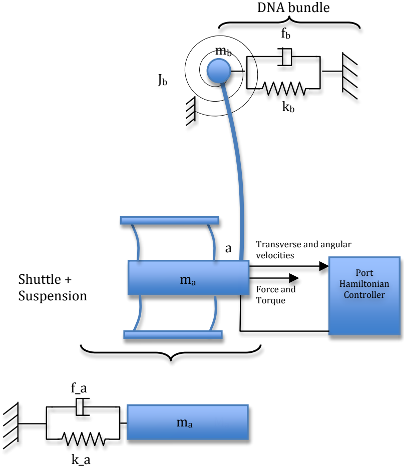
5. Conclusion and future work

Example : DNA manipulation





Example : DNA manipulation



- **PDE+ODE, BCS**
- **Casimirs**
- **Asymptotically stable,**

FIGURE: Simplified model

Example : DNA manipulation

Timoshenko beam

$$\rho(z)\frac{\partial^2 w}{\partial t^2}(z,t) = \frac{\partial}{\partial z}\left[K(z)\left(\frac{\partial w}{\partial z}(z,t) - \phi(z,t)\right)\right], \qquad z \in (a,b),\ t \geq 0,$$

$$I_\rho(z)\frac{\partial^2 \phi}{\partial t^2}(z,t) = \frac{\partial}{\partial z}\left(EI(z)\frac{\partial \phi}{\partial z}(z,t)\right) + K(z)\left(\frac{\partial w}{\partial z}(z,t) - \phi(z,t)\right),$$

Port-Hamiltonian model

$$\frac{\partial}{\partial t}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{P_1} \frac{\partial}{\partial z}\begin{bmatrix} Kx_1 \\ \frac{1}{\rho}x_2 \\ EIx_3 \\ \frac{1}{I_\rho}x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{P_0} \begin{bmatrix} Kx_1 \\ \frac{1}{\rho}x_2 \\ EIx_3 \\ \frac{1}{I_\rho}x_4 \end{bmatrix}$$

$$E = \frac{1}{2}\int_a^b \left(Kx_1^2 + \frac{1}{\rho}x_2^2 + EIx_3^2 + \frac{1}{I_\rho}x_4^2\right) dz = \frac{1}{2}\|x\|_{\mathcal{L}}^2$$

Example : DNA manipulation

Boundary port variables :

$$\begin{bmatrix} f_{\partial,\mathcal{L}x} \\ e_{\partial,\mathcal{L}x} \end{bmatrix} = \begin{bmatrix} (\rho^{-1}x_2)(b) - (\rho^{-1}x_2)(a) \\ (Kx_1)(b) - (Kx_1)(a) \\ (I_\rho^{-1}x_4)(b) - (I_\rho^{-1}x_4)(a) \\ (EIx_3)(b) - (EIx_3)(a) \\ (\rho^{-1}x_2)(b) + (\rho^{-1}x_2)(a) \\ (Kx_1)(b) + (Kx_1)(a) \\ (I_\rho^{-1}x_4)(b) + (I_\rho^{-1}x_4)(a) \\ (EIx_3)(b) + (EIx_3)(a) \end{bmatrix}.$$

Interconnexion with DNA bundle at point b and full actuation at point a

$$u = \begin{bmatrix} v(b) & \omega(b) & -v(a) & -\omega(a) \end{bmatrix},$$

$$y = \begin{bmatrix} \Gamma(b) & T(b) & F(a) & T(a) \end{bmatrix},$$

$u = W \begin{bmatrix} f_{\partial,\mathcal{L}x} \\ e_{\partial,\mathcal{L}x} \end{bmatrix}$, $y = \tilde{W} \begin{bmatrix} f_{\partial,\mathcal{L}x} \\ e_{\partial,\mathcal{L}x} \end{bmatrix}$ where

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$\tilde{W} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Finite dimensional system

- At point b : $v_b = (x_b, m_b\dot{x}_b, \theta_b, m_b\omega_b)^T$, $u_b = \begin{bmatrix} F(b) & T(b) \end{bmatrix}^T$ and $y_b = \begin{bmatrix} v(b) & \omega(b) \end{bmatrix}^T$:

$$\dot{v}_b = (J_b - R_b)\frac{dE_b}{dv_b} + g_b u_b, \qquad y_b = g_b^T \frac{dE_b}{dv_b}$$

with $E_b(x_b, m_b\dot{x}_b, \theta_b, J_b\omega_b) = \frac{k_b}{2}x_b^2 + \frac{1}{2M}(M\dot{x}_{c2})^2 + \frac{k_{\theta_b}}{2}\theta_b^2 + \frac{1}{2J_b}(J_b\omega_b)^2$ and

$$J_b = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \; R_b = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & f_b & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_{\theta_b} \end{bmatrix}, \; g_b^T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

- At point a :

$$\dot{v}_a = (J_a - R_a)\frac{dE_a}{dv_a} + g_a u_a, \qquad y_a = g_a^T \frac{dE_a}{dv_a}$$

with $E_a(x_a, m_a\dot{x}_a) = \frac{1}{2}(-k + k_c)\,x_a^2 + \frac{1}{2m_b}(m_a\dot{x}_a)^2$ and

$$J_a = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \; R_b = \begin{bmatrix} 0 & 0 \\ 0 & f_a + f_c \end{bmatrix}, \; g_a^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Casimirs

$$C(x,v) = \kappa = \Gamma^T v + \int_a^b \Psi(z,t)^T x(z,t) dz$$

satisfy :

- from condition (3) :

$$\begin{aligned} \Psi_1 &= C_1 \\ \Psi_2 &= C_4 z + C_2 \\ \Psi_3 &= -C_1 z + C_3 \\ \Psi_4 &= C_4 \end{aligned}$$

  where $C_i, i \in [1, \cdots, 4]$ are constants.

- from condition (4) :

$$\Gamma_2 = \Gamma_4 = \Gamma_6 = 0 \quad (6)$$

$$\Gamma_1 = -\Psi_1(b) \quad (7)$$

$$\Gamma_3 = \Psi_3(b) \quad (8)$$

$$\Gamma_5 = -\Psi_1(a) \quad (9)$$

Casimirs

- from condition (5) :

$$\Gamma_2 = -\Psi_2(b) \tag{10}$$

$$\Gamma_4 = -\Psi_4(b) \tag{11}$$

$$\Gamma_6 = -\Psi_2(a) \tag{12}$$

$$\Psi_3(a) = 0 \tag{13}$$

Then the Casimir functions are defined as :

$$\kappa = -C_1 x_b - C_1(a+b)\Theta_b + C_1 x_a + \int_a^b C_1\left(x_1 - (z+a)x_3\right) dz \tag{14}$$

Control design

The goal of the control law is to shape the total energy $E_d(v_b, x, v_a)$ such that it presents a minimum in the desired position of the tip of the arm, *i.e.* :

$$x_b^* = x_{b,c}^* \text{ and} \dot{x}_b^* = 0, \dot{\theta_b}^* = 0, \phi_a^* = 0, \dot{\phi}_a^* = 0.$$

The degrees of freedom we use for control design are the programmable "stiffness" and "damping" k_c and f_c.
From the equilibrium conditions and Casimirs we compute

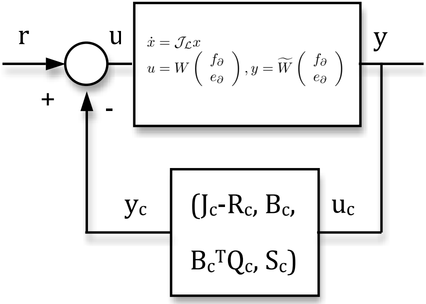
$$k_c(x_b^*)$$

and then :

$$F_c = -k_c(x_b^*)x_a - f_c\dot{x}_a$$

Outline

Exponential stability : assumptions



Exponential stability : assumptions

Finite dimensional port Hamiltonian controller

$$\dot{v} = (J_c - R_c)Q_c v + B_c u_c, \quad y_c = B_c^\top Q_c v + S_c u_c, \quad E_c(t) = \frac{1}{2} v(t)^\top Q_c v(t)$$

where we assume that $Q_c = Q_c^\top > 0$, $J_c = -J_c^\top$, $R_c = R_c^\top \geq 0$, $S_c = S_c^\top > 0$ and B_c are real constant matrices of proper dimensions. Furthermore, the controller is assumed to be exponentially stable, i.e., $A_c := (J_c - R_c)Q_c$ is Hurwitz.
The system is a strictly input passive port-Hamiltonian system, i.e. there exists a $\sigma > 0$ such that

$$\dot{E}_c(t) \leq u_c(t)^\top y_c(t) - \sigma \|u_c(t)\|^2.$$

Input and output of the BCS

We also assume that the BCS satisfies

$$\|u(t)\|^2 + \|y(t)\|^2 \geq \epsilon \|\mathcal{L}x(t,b)\|^2 \tag{15}$$

for some $\epsilon > 0$.

Exponential stability [Ramirez and Le Gorrec, 2013]

The proof follows the same steps as in [Villegas et al., 2009] including the energy contribution of the finite dimensional controller :

Exponential stability [Ramirez and Le Gorrec, 2013]

The proof follows the same steps as in [Villegas et al., 2009] including the energy contribution of the finite dimensional controller :

Lemma

Consider the controlled BCS with $r(t) = 0$, for all $t \geq 0$. Due to the contraction property the energy of the system $\tilde{E}(t) = \frac{1}{2}\|x(t)\|^2_{\mathcal{L}} + \frac{1}{2}v(t)^T Q_c v(t)$ satisfies for τ large enough

$$\tilde{E}(\tau) \leq c(\tau)\int_0^\tau \|(\mathcal{L}x)(t,b)\|^2 dt + \frac{2c(\tau)}{c_1}\int_0^\tau E_c(t)dt,$$

$$\tilde{E}(\tau) \leq c(\tau)\int_0^\tau \|(\mathcal{L}x)(t,a)\|^2 dt + \frac{2c(\tau)}{c_1}\int_0^\tau E_c(t)dt,$$

where c is a positive constant that only depends on τ and c_1 a positive constant.

Exponential stability [Ramirez and Le Gorrec, 2013]

The proof follows the same steps as in [Villegas et al., 2009] including the energy contribution of the finite dimensional controller :

Lemma

Consider the controlled BCS with $r(t) = 0$, for all $t \geq 0$. Due to the contraction property the energy of the system $\tilde{E}(t) = \frac{1}{2}\|x(t)\|^2_{\mathcal{L}} + \frac{1}{2}v(t)^T Q_c v(t)$ satisfies for τ large enough

$$\tilde{E}(\tau) \leq c(\tau) \int_0^\tau \|(\mathcal{L}x)(t,b)\|^2 dt + \frac{2c(\tau)}{c_1} \int_0^\tau E_c(t)dt,$$
$$\tilde{E}(\tau) \leq c(\tau) \int_0^\tau \|(\mathcal{L}x)(t,a)\|^2 dt + \frac{2c(\tau)}{c_1} \int_0^\tau E_c(t)dt,$$

where c is a positive constant that only depends on τ and c_1 a positive constant.

Theorem

Consider the BCS previously defined with $r(t) = 0$, for all $t \geq 0$. It is exponentially stable as soon as the finite dimensional boundary controller is exponentially stable and strictly input passive and $\|u(t)\|^2 + \|y(t)\|^2 \geq \epsilon\|\mathcal{L}x(t,b)\|^2, \ \epsilon > 0$ $\Leftarrow$ we used the Lemma, the total energy as Lyapunov function, and the properties of the controller

Proof (1)

In order to prove the exponential stability we need the following lemmas

Lemma

There exist strictly positive constants κ_2, κ_3 *and* κ_4 *such that for all* $\tau > 0$ *the energy of the PH controller satisfies :*

$$E_C(\tau) \leq \kappa_1(\tau) E_C(0) + \kappa_3 \int_0^\tau \|u_C(t)\|^2 dt \quad (16)$$

where $\kappa_1(\tau) = \kappa_4 e^{-\kappa_2 \tau}$.

Lemma

There exists positive constants ξ_1, ξ_2 *and* τ_0 *such for all* $\tau > \tau_0$ *the energy of the PH controller satisfies*

$$\int_0^\tau E_C(t) dt \leq \xi_1 \int_0^\tau v^\top(t) Q_C R_C Q_C v(t) dt + \xi_2 \int_0^\tau \|u_C(t)\|^2 dt$$

Lemma

For every $\delta_1 > 0$ *there exists a* $\delta_2 > 0$ *such that for all* $\tau > 0$ *the energy of the PH controller satisfies the relation*

$$\int_0^\tau \delta_1 E_C(t) + \|y_C(t)\|^2 dt \leq \delta_2 \int_0^\tau E_C(t) + \|u_C(t)\|^2 dt. \quad (17)$$

Proof (2)

Let $\sigma > 0$ be such that $S_c \geq \sigma I$. The time derivative of the total energy satisfies

$$\begin{aligned}\dot{\tilde{E}} &= -v^\top Q_c R_c Q_c v - u_c^\top S_c u_c \\ &\leq -v^\top Q_c R_c Q_c v - \sigma u_c^\top u_c, \qquad \text{since } S_c \geq \sigma I \\ &= -v^\top Q_c R_c Q_c v - \sigma\epsilon_1 u_c^\top u_c - \sigma\epsilon_2 u_c^\top u_c \\ &= -v^\top Q_c R_c Q_c v - \sigma\epsilon_1 \|u_c\|^2 - \sigma\epsilon_2 \left(\|y\|^2 + \|u\|^2\right) + \\ &\quad \sigma\epsilon_2 \|u\|^2\end{aligned}$$

with $\epsilon_1 + \epsilon_2 = 1$ and where we have used that $u_c = -y$.

Proof (3)

Using our main Assumption we have

$$\dot{\tilde{E}} \leq -v^\top Q_c R_c Q_c v - \sigma\epsilon_1 \|u_c\|^2 - \sigma\epsilon_2\epsilon \|\mathcal{L}x(t,b)\|^2 + \sigma\epsilon_2 \|y_c\|^2.$$

Integrating this equation on $t \in [0, \tau]$ we have

$$\tilde{E}(\tau) - \tilde{E}(0) \leq -\int_0^\tau v^\top(t) Q_c R_c Q_c v(t) dt$$
$$+ \int_0^\tau -\sigma\epsilon_1 \|u_c(t)\|^2 - \sigma\epsilon_2\epsilon \|\mathcal{L}x(t,b)\|^2 + \sigma\epsilon_2 \|y_c(t)\|^2 dt.$$

Next choose τ sufficiently large such that Lemmas 2 and 3 hold. Using the latter lemma we have

$$\tilde{E}(\tau) - \tilde{E}(0) \leq -\int_0^\tau v^\top Q_c R_c Q_c v + \sigma\epsilon_1 \|u_c\|^2 dt$$
$$+ \frac{\sigma\epsilon_2\epsilon}{c(\tau)} \left(\frac{2c(\tau)}{c_1} \int_0^\tau E_c(t) dt - \tilde{E}(\tau) \right) + \sigma\epsilon_2 \int_0^\tau \|y_c\|^2 dt.$$

Grouping terms we have that

$$\tilde{E}(\tau) \left(1 + \frac{\sigma\epsilon_2\epsilon}{c(\tau)} \right) - \tilde{E}(0) \leq$$
$$-\int_0^\tau v(t)^\top Q_c R_c Q_c v(t) dt - \sigma\epsilon_1 \int_0^\tau \|u_c(t)\|^2 dt$$
$$+\sigma\epsilon_2 \left(\int_0^\tau \frac{2\epsilon}{c_1} E_c(t) + \|y_c(t)\|^2 dt \right).$$

Proof (4)

Using Lemma 3 with $\delta_1 = \frac{2\epsilon}{c_1}$ we have

$$\tilde{E}(\tau)\left(1 + \frac{\sigma\epsilon_2\epsilon}{c(\tau)}\right) - \tilde{E}(0) \leq -\int_0^\tau v(t)^\top Q_c R_c Q_c v(t)dt$$
$$+ \sigma\epsilon_2\delta_2 \int_0^\tau E_c(t)dt + \sigma(\epsilon_2\delta_2 - \epsilon_1)\int_0^\tau \|u_c(t)\|^2 dt. \quad (18)$$

Now, using Lemma 2 we obtain

$$\tilde{E}(\tau)\left(1 + \frac{\sigma\epsilon_2\epsilon}{c(\tau)}\right) - \tilde{E}(0) \leq$$
$$(\sigma\epsilon_2\delta_2\xi_1 - 1)\int_0^\tau v(t)^\top Q_c R_c Q_c v(t)dt +$$
$$\sigma(\epsilon_2\delta_2(1+\xi_2) - \epsilon_1)\int_0^\tau \|u_c(t)\|^2 dt.$$

Since ϵ_2 may be chosen to be arbitrarily small, i.e, $\epsilon_2 \ll 1$ and since $\epsilon_1 = 1 - \epsilon_2$, we finally have that $\tilde{E}(\tau) \leq c_2\tilde{E}(0)$ with $c_2 = \frac{1}{\left(1+\frac{\sigma\epsilon_2\epsilon}{c(\tau)}\right)} < 1$ which proves the theorem.

Example : DNA manipulation - extension

- Discretization scheme in order to preserve the structure of the model.
- Measurement and control at point a (static feedback on the velocity).

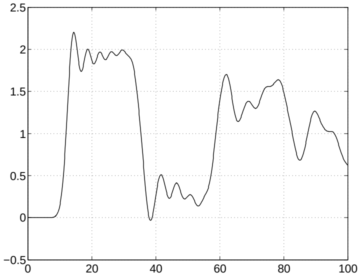


FIGURE: Open loop simulation

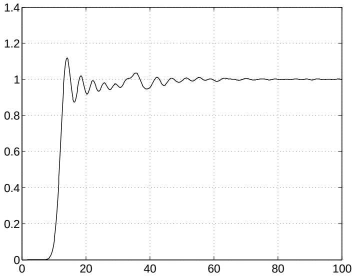


FIGURE: Closed loop simulation

Example : DNA manipulation - extension

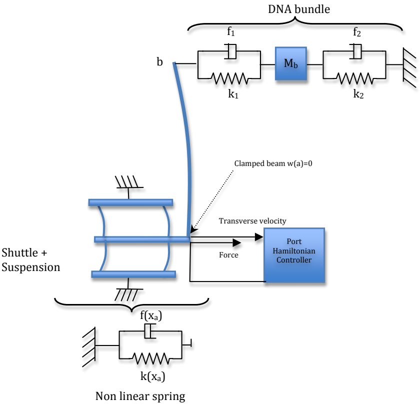


FIGURE: Simplified model

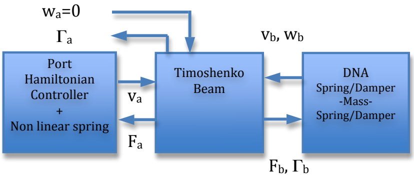


FIGURE: Control diagram

- **PDE+ODE**
- **BCS**
- **Exponentially stable,**

Outline

Conclusion and future work

- A large class of boundary control system are exponentially stable if they are interconnected in a power preserving manner with an input strictly passive and exponentially stable finite dimensional linear controller.
- We have extended the exponential stability proof of [Villegas et al., 2009] for static control of BCS for the case of dynamic boundary control.
- The approach has been illustrated on the physical example of a partially actuated micro-gripper for DNA manipulation.

Conclusion and future work

- A large class of boundary control system are exponentially stable if they are interconnected in a power preserving manner with an input strictly passive and exponentially stable finite dimensional linear controller.
- We have extended the exponential stability proof of [Villegas et al., 2009] for static control of BCS for the case of dynamic boundary control.
- The approach has been illustrated on the physical example of a partially actuated micro-gripper for DNA manipulation.

Ongoing and future work

- Extend the results to the use of non-linear boundary controllers.
- Include the interaction with the liquid medium and liquid meniscus.
- Perform Energy Shaping methods.
- Generalization to 2D and 3D cases.

Thank you for your attention !

Le Gorrec, Y., Zwart, H., and Maschke, B. (2005).
Dirac structures and boundary control systems associated with skew-symmetric differential operators.
SIAM Journal on Control and Optimization, 44(5) :1864–1892.

Ramirez, H. and Le Gorrec, Y. (2013).
Exponential stability of a class of pde's with dynamic boundary control.
In *American Control Conference*, Washington, USA.

Villegas, J., Zwart, H., Le Gorrec, Y., and Maschke, B. (2009).
Exponential stability of a class of boundary control systems.
IEEE Transactions on Automatic Control, 54 :142–147.

Villegas, J. A. (2007).
A port-Hamiltonian Approach to Distributed Parameter Systems.
PhD thesis, Universiteit Twente.