Erratum: Standing Waves for Nonlinear Schrödinger Equations with a General Nonlinearity

Jaeyoung Byeon and Louis Jeanjean

June 20, 2008

Erratum: Arch. Rational Mech. Anal. DOI 10.1007/s00205-006-0019-3

Through answering to some questions from Joao Marcos Bezzerra do O on the paper above, we found some mistakes in the paper. We thank him for his interest and careful reading. Here we correct these mistakes.

First at the beginning of the proof of Proposition 4, 14-17 line on page 193, the correct version is:

By compactness of S_m and $\mathcal{M}^\beta$, there exist $Z \in S_m$, $\{x_\varepsilon\} \subset \mathcal{M}^\beta$ and $x \in \mathcal{M}^\beta$ with $x_\varepsilon \to x$ such that

$$\|u_\varepsilon - \varphi_\varepsilon(\cdot - x_\varepsilon/\varepsilon)Z(\cdot - x_\varepsilon/\varepsilon)\|_\varepsilon \leq 2d$$

for small $\varepsilon > 0$.

Having replaced x by x_ε in the inequality (18), we can follow the same steps in the rest of the proof of Proposition 4 as before to prove the claim of Proposition 4, since $x_\varepsilon \to x$.

Secondly, in the proof of Proposition 8, the statement "Then, it follows in a standard way that u is a critical point of Γ_ε" is problematic. To avoid having to prove directly this statement we replace respectively Proposition 7 and 8 by Proposition 1 and 2 below.

Proposition 1 *For sufficiently small $\varepsilon > 0$ and sufficiently large $R > 0$, there exists a sequence $\{u_n^R\}_{n=1}^\infty \subset X_\varepsilon^d \cap H_0^1(B(0, R/\varepsilon)) \cap \Gamma_\varepsilon^{D_\varepsilon}$ such that $\Gamma'_\varepsilon(u_n^R) \to 0$ in $H_0^1(B(0, R/\varepsilon))$ as $n \to \infty$.*

Proof. We note that we can take $R_0 > 0$ sufficiently large so that $O \subset B(0, R_0)$ and $\gamma_\varepsilon(s) \in H_0^1(B(0, R/\varepsilon))$ for any $s \in [0, 1]$, $R > R_0$ and sufficiently small $\varepsilon > 0$.

By Proposition 6 in the paper, there exists $\alpha > 0$ such that for sufficiently small $\varepsilon > 0$,

$$\Gamma_\varepsilon(\gamma_\varepsilon(s)) \geq C_\varepsilon - \alpha \quad \text{implies that} \quad \gamma_\varepsilon(s) \in H_0^1(B(0, R/\varepsilon)) \cap X_\varepsilon^{d/2}.$$

If Proposition 1 does not hold for sufficiently small $\varepsilon > 0$, there exists $a_R(\varepsilon) > 0$ such that $|\Gamma'_\varepsilon(u)| \geq a_R(\varepsilon)$ on $H_0^1(B(0, R/\varepsilon)) \cap X_\varepsilon^d \cap \Gamma_\varepsilon^{D_\varepsilon}$. Note that any $u \in H_0^1(B(0, R/\varepsilon))$ can be regarded as an element in H_ε by defining $u = 0$ on $\mathbf{R}^N \setminus B(0, R/\varepsilon)$. Then, by using a pseudo-gradient flow in $H_0^1(B(0, R/\varepsilon))$ and following the same scheme in the original proof, we get a contradiction. This completes the proof. □

Proposition 2 *For sufficiently small fixed $\varepsilon > 0$, Γ_ε has a critical point $u_\varepsilon \in X_\varepsilon^d \cap \Gamma_\varepsilon^{D_\varepsilon}$.*

Proof. Let $\varepsilon > 0$ be fixed and sufficiently small. Let $\{u_n^R\}_{n=1}^\infty \subset H_0^1(B(0, R/\varepsilon))$ be a Palais-Smale sequence as given by Proposition 1. Since $\{u_n^R\}_{n=1}^\infty$ is bounded in $H_0^1(B(0, R/\varepsilon))$, we deduce from the compactness of the imbedding $H_0^1(B(0, R/\varepsilon)) \hookrightarrow L^{p+1}(B(0, R/\varepsilon))$ that u_n^R converges, up to a subsequence, strongly to some u^R in $H_0^1(B(0, R/\varepsilon))$ and that u^R is a critical point of Γ_ε on $H_0^1(B(0, R/\varepsilon))$. Thus, $u^R \in H_0^1(B(0, R/\varepsilon))$ satisfies

$$\Delta u^R - V_\varepsilon u^R + f(u^R) = (p+1)\Big(\int \chi_\varepsilon (u^R)^2 dx - 1\Big)_+^{\frac{p-1}{2}} \chi_\varepsilon u^R \ \text{ in } \ B(0, R/\varepsilon). \quad (1)$$

Since $f(t) =$ for $t \leq 0$, we see that $u^R > 0$ in $B(0, R/\varepsilon)$ and it follows that

$$\Delta u^R - V_\varepsilon u^R + f(u^R) \geq 0 \ \text{ in } \ B(0, R/\varepsilon). \quad (2)$$

Note that $\{\|u^R\|_\varepsilon\}_{R \geq R_0}$ and $\{\Gamma_\varepsilon(u^R)\}_R$ are uniformly bounded for small $\varepsilon > 0$. Then, $\{Q_\varepsilon(u^R)\}_R$ is uniformly bounded for small $\varepsilon > 0$, and from standard elliptic estimates we see that$\{u^R\}$ is bounded in L^∞ uniformly for small $\varepsilon > 0$. Then, since $\{Q_\varepsilon(u^R)\}_R$ is uniformly bounded for small $\varepsilon > 0$, we see from elliptic estimates that for sufficiently small $\varepsilon > 0$, $|f(u^R(x))| \leq \frac{1}{2}V(\varepsilon x)u^R(x)$ if $|x| \geq 2R_0$. Applying a comparison principle to (2), we see that for some $C, c > 0$, independent of $R > R_0$,

$$u^R(x) \leq C\exp(-(|x| - 2R_0)). \tag{3}$$

Then, we see from (2) and (3) that

$$\lim_{A\to\infty} \int_{\mathbf{R}^N\setminus B(0,A)} |\nabla u^R|^2 + (u^R)^2 dx = 0 \quad \text{uniformly for large } \ R > R_0. \tag{4}$$

Since $\{u^R\}_R$ is bounded in H_ε, we may assume that u^R converges weakly to some u_ε in H_ε as $R \to \infty$. Then, since u^R is a solution of (1), we see from (3) and (4) that u^R converges strongly to $u_\varepsilon \in X_\varepsilon^d \cap \Gamma_\varepsilon^{D_\varepsilon}$ and that

$$\Delta u_\varepsilon - V_\varepsilon u_\varepsilon + f(u_\varepsilon) = (p+1)\Big(\int \chi_\varepsilon u_\varepsilon^2 dx - 1\Big)_+^{\frac{p-1}{2}} \chi_\varepsilon u_\varepsilon \ \text{ in } \ \mathbf{R}^N.$$

This prove the claim. □