Discretizations of fractional powers of the Laplacian in bounded domains

Félix del Teso ***(NTNU)***

Joint work with:

Nicole Cusimano *(BCAM)* Luca Gerardo-Giorda *(BCAM)* Gianni Pagnini *(BCAM)*

Abstract

The fractional Laplacian is a differential operator of non-integer order that has been extensively studied in the last few decades and is naturally defined on the whole $\mathbb{R}^N$. As many other fractional order derivatives and integrals, this operator has been often used to model transport processes which generalize classical Brownian motion. However, many physical problems of interest are defined in bounded domains and the use of the fractional Laplacian as modeling tool in this context poses the challenge of providing a meaningful interpretation of the operator in these settings.

Following the heat semi-group formalism, we consider a family of operators which are boundary conditions dependent and discuss a suitable approach for their numerical discretizations by combining quadratures rules with finite element methods.

This approach will provide a flexible strategy for numerical computations of fractional powers of operators in bounded settings with different homogeneous boundary conditions in multi-dimensional (possibly irregular) domains. We also discuss the corresponding fractional Poisson problem and provide a natural way of defining different types of non-homogeneous boundary conditions.