Nonlinear Schrödinger equations in the presence of an external magnetic field: semiclassical limit and other properties

Joint works with Denis Bonheure, Silvia Cingolani and Jean Van-Schaftingen.

In this talk, I will present some results about a nonlinear Schrödinger equations in the presence of an external magnetic field

$$-(\hbar\nabla + iA)^2 u + Vu = |u|^{p-2}u, \qquad \text{in } \mathbb{R}^N.$$

Here $A : \mathbb{R}^N \to \mathbb{R}^N$ is the magnetic potential, and the magnetic operator is defined as

$$-(\hbar\nabla + iA)^2 u = -\hbar^2\Delta u - 2i\hbar A \cdot \nabla u - i\hbar\nabla \cdot Au + |A|^2 u.$$

First of all, I will consider this equation in $\mathbb{R}^3$ (but this could be extended) when V and A satisfy some cylindrical symmetry properties. Under those assumptions, we can find a symmetric solution $u_\hbar$ which concentrates on a circle (due to the cylindrical symmetry) at the semiclassical limit $\hbar \to 0$. Moreover, the radius of that circle depends on V and A, unlike the previous results in the literature, where they found that the location of the concentration was only driven by V.

Next, I will restrict myself to the linear magnetic potentials (or equivalently to constant magnetic fields), and study some uniqueness, symmetry and decay properties of the groundstate when A is small (in that case, we can use the known properties of the limit problem with $A = 0$).