SOLITARY WAVES WITH PRESCRIBED L²-MASS FOR CUBIC SCHRÖDINGER EQUATIONS AND SYSTEMS ON BOUNDED DOMAINS

Given m > 0, we study the problem

find
$$(u,\omega) \in H_0^1(\Omega) \times \mathbb{R}$$
 such that
$$\begin{cases} -\Delta u + \omega u = u^p \\ \|u\|_{L^2(\Omega)}^2 = m, \\ u > 0, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded regular domain and p is Sobolev-subcritical, that is to say p+1 < 2N/(N-2). Such problem arises in the search of solitary wave solutions for nonlinear Schrödinger equations with power nonlinearity on bounded domains.

In the case Ω is a radial domain and $p \leq 1 + 4/N$, we give a full picture of the existence of solutions and of the orbital stability of the associated standing waves, thus proving a conjecture raised in [Fibich, Merle, Phys. D, 2011]. We have partial results in the case p > 1 + 4/N and in non-radial domains.

We address a similar question for systems of coupled Schrödinger equations, obtaining results for small masses.

The talk is based on two papers in collaboration with G. Verzini (Politecnico di Milano) and H. Tavares (Instituto Superior Técnico de Lisboa).

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